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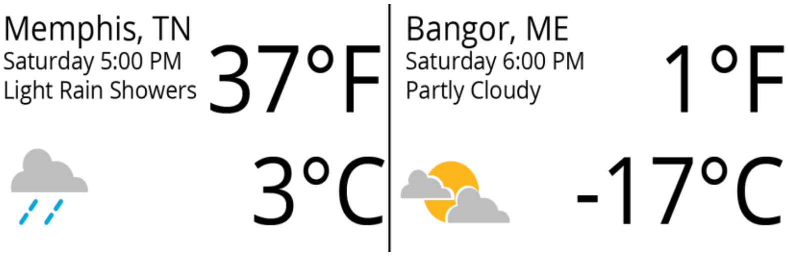
Unit 7, Lesson 1: Positive and Negative Numbers

Let's explore how we represent temperatures and elevations.

1.1: Notice and Wonder: Memphis and Bangor



What do you notice?

What do you wonder?

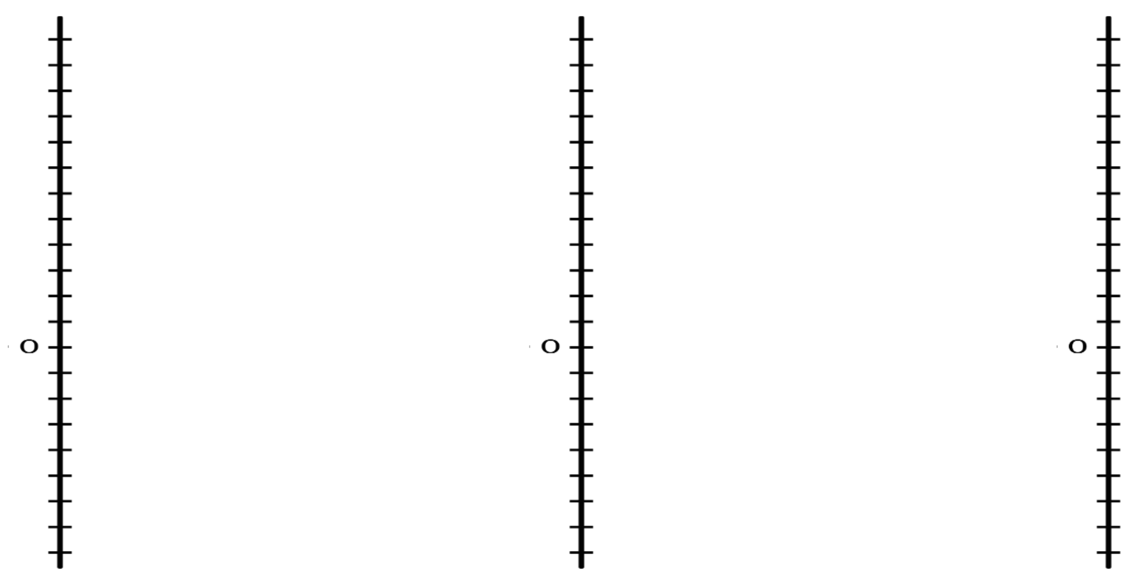
1.2: Above and Below Zero

1. Here are three situations involving changes in temperature and three number lines. Represent each change on a number line. Then, answer the question.

    a. At noon, the temperature was 5 degrees Celsius. By late afternoon, it has risen 6 degrees Celsius. What was the temperature late in the afternoon?

    b. The temperature was 8 degrees Celsius at midnight. By dawn, it has dropped 12 degrees Celsius. What was the temperature at dawn?

    c. Water freezes at 0 degrees Celsius, but the freezing temperature can be lowered by adding salt to the water. A student discovered that adding half a cup of salt to a gallon of water lowers its freezing temperature by 7 degrees Celsius. What is the freezing temperature of the gallon of salt water?



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2. Discuss with a partner:

   a. How did each of you name the resulting temperature in each situation?

   b. What does it mean when the temperature is above 0? Below 0?

   c. Do numbers less than 0 make sense in other contexts? Give some specific examples to show how they do or do not make sense.

1.3: High Places, Low Places

1. Here is a table that shows elevations of various cities.

city	elevation (feet)
Harrisburg, PA	320
Bethell, IN	1,211
Denver, CO	5,280
Coachella, CA	-22
Death Valley, CA	-282
New York City, NY	33
Miami, FL	0

   a. On the list of cities, which city has the second highest elevation?

   b. How would you describe the elevation of Coachella, CA in relation to sea level?

   c. How would you describe the elevation of Death Valley, CA in relation to sea level?

   d. If you are standing on a beach right next to the ocean, what is your elevation?

   e. How would you describe the elevation of Miami, FL?

   f. A city has a higher elevation than Coachella, CA. Select all numbers that could represent the city's elevation. Be prepared to explain your reasoning.

-11 feet -35 feet 4 feet -8 feet 0 feet

2. Here are two tables that show the elevations of highest points on land and lowest points in the ocean. Distances are measured from sea level.

mountain	continent	elevation (meters)
Everest	Asia	8,848
Kilimanjaro	Africa	5,895
Denali	North America	6,168
Pikchu Pikchu	South America	5,664

trench	ocean	elevation (meters)
Mariana Trench	Pacific	-11,033
Puerto Rico Trench	Atlantic	-8,600
Tonga Trench	Pacific	-10,882
Sunda Trench	Indian	-7,725

a. Which point in the ocean is the lowest in the world? What is its elevation?

b. Which mountain is the highest in the world? What is its elevation?

c. If you plot the elevations of the mountains and trenches on a vertical number line, what would 0 represent? What would points above 0 represent? What about points below 0?

d. Which is farther from sea level: the deepest point in the ocean, or the top of the highest mountain in the world? Explain.

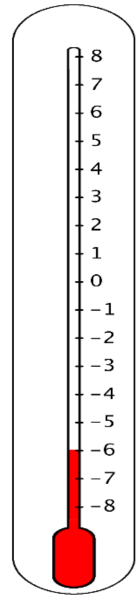
Are you ready for more?

A spider spins a web in the following way:

- It starts at sea level.
- It moves up one inch in the first minute.
- It moves down two inches in the second minute.
- It moves up three inches in the third minute.
- It moves down four inches in the fourth minute.

Assuming that the pattern continues, what will the spider's elevation be after an hour has passed?

Lesson 1 Summary

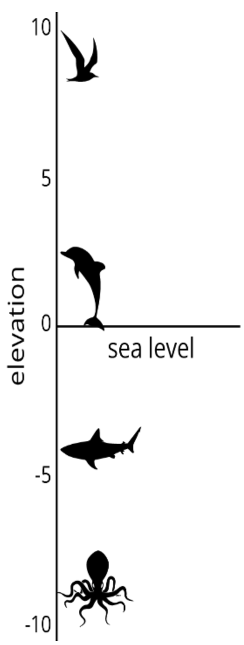


Positive numbers are numbers that are greater than 0. **Negative numbers** are numbers that are less than zero. The meaning of a negative number in a context depends on the meaning of zero in that context.

For example, if we measure temperatures in degrees Celsius, then 0 degrees Celsius corresponds to the temperature at which water freezes.

In this context, positive temperatures are warmer than the freezing point and negative temperatures are colder than the freezing point. A temperature of -6 degrees Celsius means that it is 6 degrees away from 0 and it is less than 0. This thermometer shows a temperature of -6 degrees Celsius.

If the temperature rises a few degrees and gets very close to 0 degrees without reaching it, the temperature is still a negative number.



Another example is elevation, which is a distance above or below sea level. An elevation of 0 refers to the sea level. Positive elevations are higher than sea level, and negative elevations are lower than sea level.

Lesson 1 Glossary Terms

negative number
positive number

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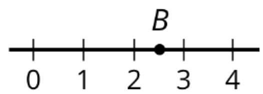
Unit 7, Lesson 2: Points on the Number Line

Let’s plot positive and negative numbers on the number line.

2.1: A Point on the Number Line



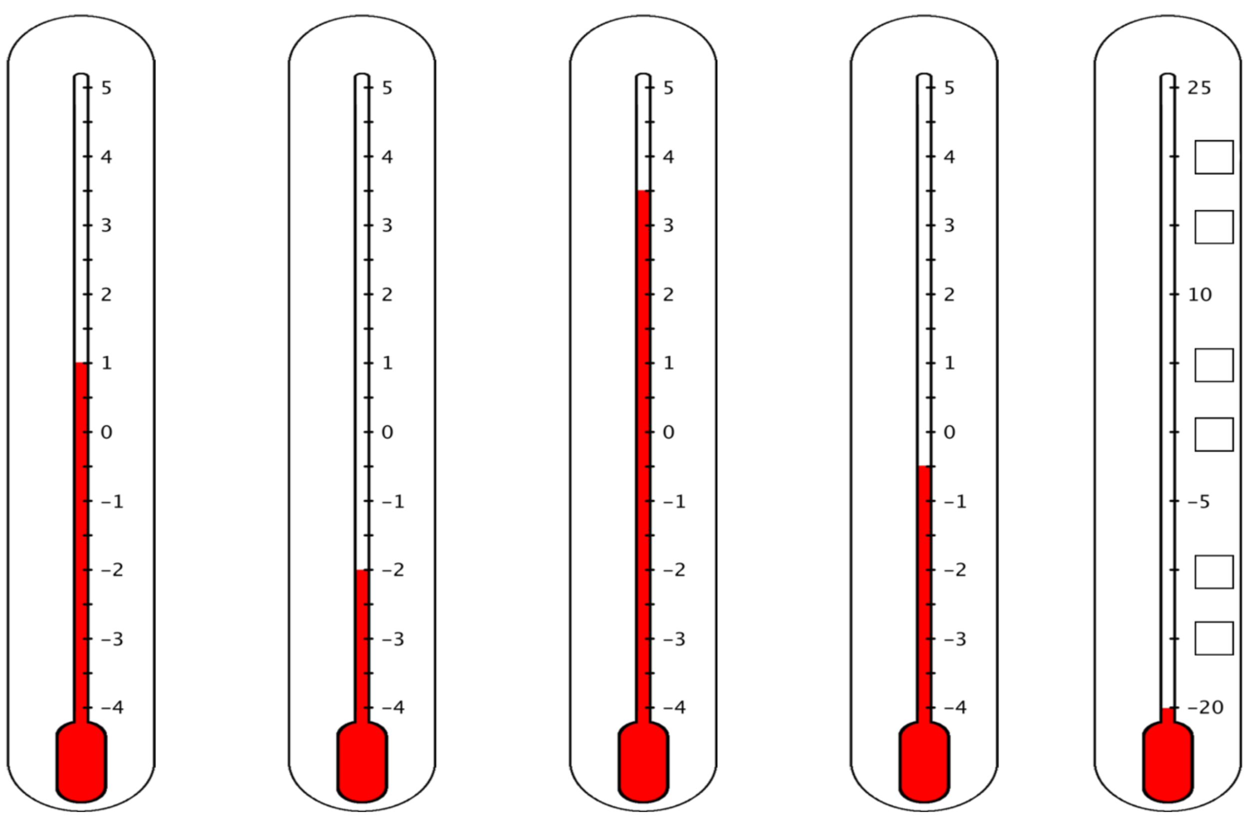
Which of the following numbers could be B?

2.5 $\frac{2}{5}$ $\frac{5}{2}$ $\frac{25}{10}$ 2.49

2.2: What’s the Temperature?

1. Here are five thermometers. The first four thermometers show temperatures in Celsius. Write the temperatures in the blanks.

a. ________ b. ________ c. ________ d. ________ e.



The last thermometer is missing some numbers. Write them in the boxes.

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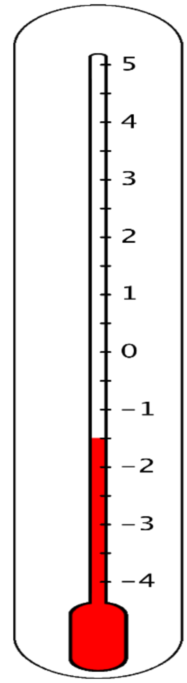
2. Elena says that the thermometer shown here reads $-2.5°C$ because the line of the liquid is above $-2°C$. Jada says that it is $-1.5°C$. Do you agree with either one of them? Explain your reasoning.



3. One morning, the temperature in Phoenix, Arizona was $8°C$ and the temperature in Portland, Maine was $12°C$ cooler. What was the temperature in Portland?

2.3: Folded Number Lines

Your teacher will give you a sheet of tracing paper on which to draw a number line.

1. Follow the steps to make your own number line.
   - Use a straightedge or a ruler to draw a horizontal line. Mark the middle point of the line and label it 0.
   - To the right of 0, draw tick marks that are 1 centimeter apart. Label the tick marks 1, 2, 3. . . 10. This represents the positive side of your number line.
   - Fold your paper so that a vertical crease goes through 0 and the two sides of the number line match up perfectly.
   - Use the fold to help you trace the tick marks that you already drew onto the opposite side of the number line. Unfold and label the tick marks -1, -2, -3. . . -10. This represents the negative side of your number line.
2. Use your number line to answer these questions:

   a. Which number is the same distance away from zero as is the number 4?

   b. Which number is the same distance away from zero as is the number -7?

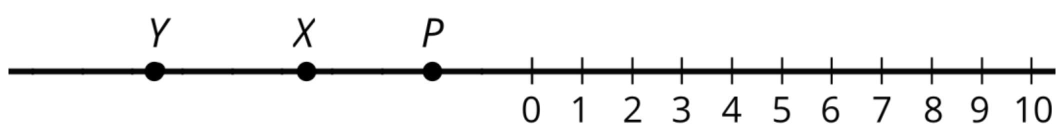
c. Two numbers that are the same distance from zero on the number line are called **opposites.** Find another pair of opposites on the number line.

d. Determine how far away the number 5 is from 0. Then, choose a positive number and a negative number that is each farther away from zero than is the number 5.

e. Determine how far away the number -2 is from 0. Then, choose a positive number and a negative number that is each farther away from zero than is the number -2.

Pause here so your teacher can review your work.

3. Here is a number line with some points labeled with letters. Determine the location of points P, X, and Y.



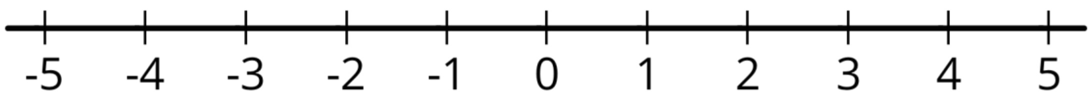
If you get stuck, trace the number line and points onto a sheet of tracing paper, fold it so that a vertical crease goes through 0, and use the folded number line to help you find the unknown values.

Are you ready for more?

At noon, the temperatures in Portland, Maine and Phoenix, Arizona had opposite values. The temperature in Portland was 18°C lower than in Phoenix. What was the temperature in each city? Explain your reasoning.

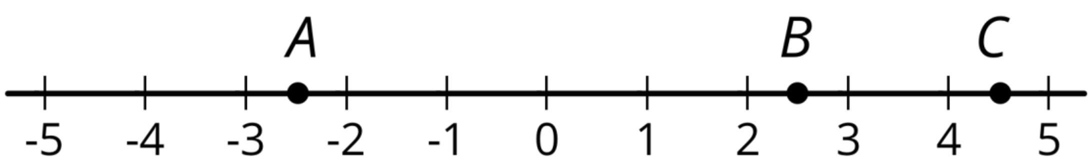
Lesson 2 Summary

Here is a number line labeled with positive and negative numbers. The number 4 is positive, so its location is 4 units to the right of 0 on the number line. The number -1.1 is negative, so its location is 1.1 units to the left of 0 on the number line.



We say that the *opposite* of 8.3 is -8.3, and that the *opposite* of $\frac{-3}{2}$ is $\frac{3}{2}$. Any pair of numbers that are equally far from 0 are called **opposites**.

Points A and B are opposites because they are both 2.5 units away from 0, even though A is to the left of 0 and B is to the right of 0.



A positive number has a negative number for its opposite. A negative number has a positive number for its opposite. The opposite of 0 is itself.

You have worked with positive numbers for many years. All of the positive numbers you have seen—whole and non-whole numbers—can be thought of as fractions and can be located on a the number line.

To locate a non-whole number on a number line, we can divide the distance between two whole numbers into fractional parts and then count the number of parts. For example, 2.7 can be written as $2\frac{7}{10}$. The segment between 2 and 3 can be partitioned into 10 equal parts or 10 tenths. From 2, we can count 7 of the tenths to locate 2.7 on the number line.

All of the fractions and their opposites are what we call **rational numbers**. For example, 4, -1.1, 8.3, -8.3, $\frac{-3}{2}$, and $\frac{3}{2}$ are all rational numbers.

Lesson 2 Glossary Terms

opposite
rational number

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Unit 7, Lesson 3: Comparing Positive and Negative Numbers

Let's compare numbers on the number line.

3.1: Which One Doesn't Belong: Inequalities

Which inequality doesn't belong?

$\frac{5}{4} < 2$ $8.5 > 0.95$ $8.5 < 7$ $10.00 < 100$

3.2: Comparing Temperatures

Here are the low temperatures, in degrees Celsius, for a week in Anchorage, Alaska.

day	Mon	Tues	Weds	Thurs	Fri	Sat	Sun
temperature	5	-1	-5.5	-2	3	4	0

1. Plot the temperatures on a number line. Which day of the week had the lowest low temperature?

2. The lowest temperature ever recorded in the United States was -62 degrees Celsius, in Prospect Creek Camp, Alaska. The average temperature on Mars is about -55 degrees Celsius.

    a. Which is warmer, the coldest temperature recorded in the USA, or the average temperature on Mars? Explain how you know.

    b. Write an inequality to show your answer.

3. On a winter day the low temperature in Anchorage, Alaska was -21 degrees Celsius and the low temperature in Minneapolis, Minnesota was -14 degrees Celsius.

    Jada said: "I know that 14 is less than 21, so -14 is also less than -21. This means that it was colder in Minneapolis than in Anchorage."

    Do you agree? Explain your reasoning.

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Are you ready for more?

Another temperature scale frequently used in science is the *Kelvin scale*. In this scale, 0 is the lowest possible temperature of anything in the universe, and it is -273.15 degrees in the Celsius scale. Each 1 K is the same as 1°C, so 10 K is the same as -263.15°C.

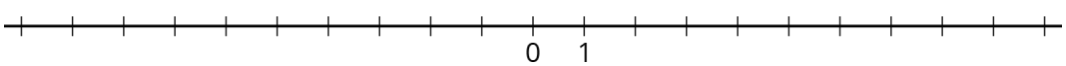
1. Water boils at 100°C. What is this temperature in K?

2. Ammonia boils at -35.5°C. What is the boiling point of ammonia in K?

3. Explain why only positive numbers (and 0) are needed to record temperature in K.

3.3: Rational Numbers on a Number Line

1. Plot the numbers -2, 4, -7, and 10 on the number line. Label each point with its numeric value.



2. Decide whether each inequality statement is true or false. Be prepared to explain your reasoning.

   $-2 < 4$ $\quad -2 < -7 \quad$ $4 > -7$ $\quad -7 > 10$

3. Andre says that $\frac{1}{4}$ is less than $-\frac{3}{4}$ because, of the two numbers, $\frac{1}{4}$ is closer to 0. Do you agree? Explain your reasoning.

4. Answer each question. Be prepared to explain how you know.

   a. Which number is greater: $\frac{1}{4}$ or $\frac{5}{4}$?

   b. Which is farther from 0: $\frac{1}{4}$ or $\frac{5}{4}$?

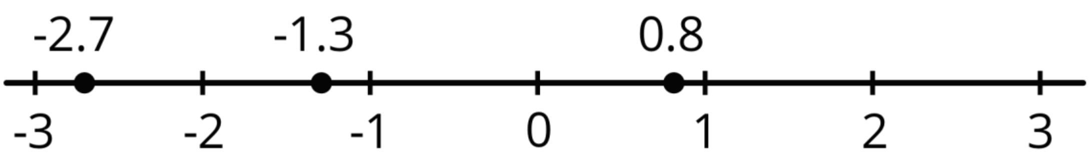
c. Which number is greater: $-\frac{3}{4}$ or $\frac{5}{8}$?

d. Which is farther from 0: $-\frac{3}{4}$ or $\frac{5}{8}$?

e. Is the number that is farther from 0 always the greater number? Explain your reasoning.

Lesson 3 Summary

We use the words *greater than* and *less than* to compare numbers on the number line. For example, the numbers -2.7, 0.8, and -1.3, are shown on the number line.



Because -2.7 is to the left of -1.3, we say that -2.7 is less than -1.3. We write:

$$-2.7 < -1.3$$

In general, any number that is to the left of a number n is less than n.

We can see that -1.3 is greater than -2.7 because -1.3 is to the right of -2.7. We write

$$-1.3 > -2.7$$

In general, any number that is to the right of a number n is greater than n

We can also see that $0.8 > -1.3$ and $0.8 > -2.7$. In general, any positive number is greater than any negative number.

Lesson 3 Glossary Terms

sign

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Unit 7, Lesson 4: Ordering Rational Numbers

Let's order rational numbers.

4.1: How Do They Compare?

Use the symbols >, <, or = to compare each pair of numbers. Be prepared to explain your reasoning.

12 ____ 19 212 ____ 190 15 ____ 1.5 9.02 ____ 9.2

6.050 ____ 6.05 0.4 ____ $\frac{9}{40}$ $\frac{19}{24}$ ____ $\frac{19}{21}$ $\frac{16}{17}$ ____ $\frac{11}{12}$

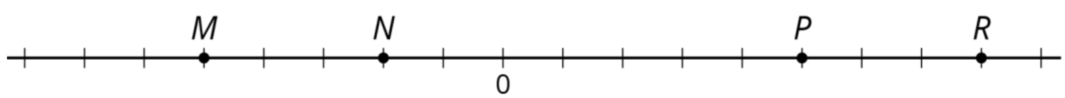
4.2: Ordering Rational Number Cards

Your teacher will give you a set of number cards. Order them from least to greatest.

Your teacher will give you a second set of number cards. Add these to the correct places in the ordered set.

4.3: Comparing Points on A Line

1.



Use each of the following terms at least once to describe or compare the values of points M, N, P, R.

- greater than
- less than
- opposite of (or opposites)
- negative number

2. Tell what the value of each point would be if:

a. P is $2\frac{1}{2}$ b. N is -0.4 c. R is 200 d. M is -15

Are you ready for more?

The list of fractions between 0 and 1 with denominators between 1 and 3 looks like this:

$$\frac{0}{1}, \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}$$

We can put them in order like this: $\frac{0}{1} < \frac{1}{3} < \frac{1}{2} < \frac{2}{3} < \frac{1}{1}$

Now let's expand the list to include fractions with denominators of 4. We won't include $\frac{2}{4}$, because $\frac{1}{2}$ is already on the list.

$$\frac{0}{1} < \frac{1}{4} < \frac{1}{3} < \frac{1}{2} < \frac{2}{3} < \frac{3}{4} < \frac{1}{1}$$

1. Expand the list again to include fractions that have denominators of 5.

2. Expand the list you made to include fractions have have denominators of 6.

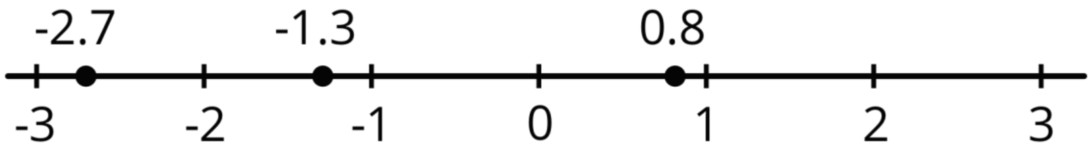
3. When you add a new fraction to the list, you put it in between two "neighbors." Go back and look at your work. Do you see a relationship between a new fraction and its two neighbors?

Lesson 4 Summary

To order rational numbers from least to greatest, we list them in the order they appear on the number line from left to right. For example, we can see that the numbers

-2.7, -1.3, 0.8

are listed from least to greatest because of the order they appear on the number line.



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Unit 7, Lesson 5: Using Negative Numbers to Make Sense of Contexts

Let's make sense of negative amounts of money.

5.1: Notice and Wonder: It Comes and Goes

activity	amount
do my chores	30.00
babysit my cousin	45.00
buy my lunch	-10.80
get my allowance	15.00
buy a shirt	-18.69
pet my dog	0.00

What do you notice? What do you wonder?

5.2: The Concession Stand

The manager of the concession stand keeps records of all of the supplies she buys and all of the items she sells. The table shows some of her records for Tuesday.

item	quantity	value in dollars
doughnuts	-58	37.70
straws	3,000	-10.35
hot dogs	-39	48.75
pizza	13	-116.87
apples	-40	14.00
french fries	-88	132.00

1. Which items did she sell? Explain your reasoning.

2. How can we interpret -58 in this situation?

3. How can we interpret -10.35 in this situation?

4. On which item did she spend the most amount of money? Explain your reasoning.

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5.3: Drinks for Sale

A vending machine in an office building sells bottled beverages. The machine keeps track of all changes in the number of bottles from sales and from machine refills and maintenance. This record shows the changes for every 5-minute period over one hour.

time	number of bottles
8:00–8:04	-1
8:05–8:09	+12
8:10–8:14	-4
8:15–8:19	-1
8:20–8:24	-5
8:25–8:29	-12
8:30–8:34	-2
8:35–8:39	0
8:40–8:40	0
8:45–8:49	-6
8:50–8:54	+24
8:55–8:59	0
service	

1. What might a positive number mean in this context? What about a negative number?

2. What would a “0” in the second column mean in this context?

3. Which numbers—positive or negative—result in fewer bottles in the machine?

4. At what time was there the greatest change to the number of bottles in the machine? How did that change affect the number of remaining bottles in the machine?

5. At which time period, 8:05–8:09 or 8:25–8:29, was there a greater change to the number of bottles in the machine? Explain your reasoning.

6. The machine must be emptied to be serviced. If there are 40 bottles in the machine when it is to be serviced, what number will go in the second column in the table?

Are you ready for more?

Priya, Mai, and Lin went to a cafe on a weekend. Their shared bill came to $25. Each student gave the server a $10 bill. The server took this $30 and brought back five $1 bills in change. Each student took $1 back, leaving the rest, $2, as a tip for the server.

As she walked away from the cafe, Lin thought, “Wait—this doesn’t make sense. Since I put in $10 and got $1 back, I wound up paying $9. So did Mai and Priya. Together, we paid $27. Then we left a $2 tip. That makes $29 total. And yet we originally gave the waiter $30. Where did the extra dollar go?”

Think about the situation and about Lin’s question. Do you agree that the numbers didn’t add up properly? Explain your reasoning.

Lesson 5 Summary

Sometimes we represent changes in a quantity with positive and negative numbers. If the quantity increases, the change is positive. If it decreases, the change is negative.

- Suppose 5 gallons of water is put in a washing machine. We can represent the change in the number of gallons as +5. If 3 gallons is emptied from the machine, we can represent the change as -3.

It is especially common to represent money we receive with positive numbers and money we spend with negative numbers.

- Suppose Clare gets $30.00 for her birthday and spends $18.00 buying lunch for herself and a friend. To her, the value of the gift can be represented as +30.00 and the value of the lunch as -18.00.

Whether a number is considered positive or negative depends on a person’s perspective. If Clare’s grandmother gives her $20 for her birthday, Clare might see this as +20, because to her, the amount of money she has increased. But her grandmother might see it as -20, because to her, the amount of money she has decreased.

In general, when using positive and negative numbers to represent changes, we have to be very clear about what it means when the change is positive and what it means when the change is negative.

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Unit 7, Lesson 6: Absolute Value of Numbers

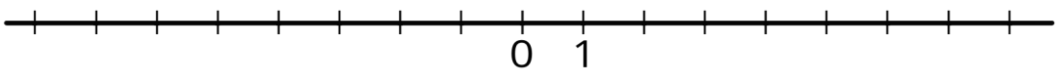
Let's explore distances from zero more closely.

6.1: Number Talk: Closer to Zero

For each pair of expressions, decide mentally which one has a value that is closer to 0.

$\frac{9}{11}$ or $\frac{15}{11}$ $\frac{1}{5}$ or $\frac{1}{9}$ 1.25 or $\frac{5}{4}$ 0.01 or 0.001

6.2: Jumping Flea



1. A flea is jumping around on a number line.

   a. If the flea starts at 1 and jumps 4 units to the right, where does it end up? How far away from 0 is this?

   b. If the flea starts at 1 and jumps 4 units to the left, where does it end up? How far away from 0 is this?

   c. If the flea starts at 0 and jumps 3 units away, where might it land?

   d. If the flea jumps 7 units and lands at 0, where could it have started?

   e. The **absolute value** of a number is the distance it is from 0. The flea is currently to the left of 0 and the absolute value of its location is 4. Where on the number line is it?

   f. If the flea is to the left of 0 and the absolute value of its location is 5, where on the number line is it?

   g. If the flea is to the right of 0 and the absolute value of its location is 2.5, where on the number line is it?

2. We use the notation $|-2|$ to say "the absolute value of -2," which means "the distance of -2 from 0 on the number line."

    a. What does $|-7|$ mean and what is its value?

    b. What does $|1.8|$ mean and what is its value?

6.3: Absolute Elevation and Temperature

1. A part of the city of New Orleans is 6 feet below sea level. We can use "-6 feet" to describe its elevation, and "$|-6|$ feet" to describe its vertical distance from sea level. In the context of elevation, what would each of the following numbers describe?

    a. 25 feet

    b. $|25|$ feet

    c. -8 feet

    d. $|-8|$ feet

2. The elevation of a city is different from sea level by 10 feet. Name the two elevations that the city could have.

3. We write "-5°C" to describe a temperature that is 5 degrees Celsius below freezing point and "5°C" for a temperature that is 5 degrees above freezing. In this context, what do each of the following numbers describe?

    a. 1°C

    b. -4°C

    c. $|12|^\circ$C

    d. $|-7|^\circ$C

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4. a. Which temperature is colder: -6°C or 3°C?

   b. Which temperature is closer to freezing temperature: -6°C or 3°C?

   c. Which temperature has a smaller absolute value? Explain how you know.

Are you ready for more?

At a certain time, the difference between the temperature in New York City and in Boston was 7 degrees Celsius. The difference between the temperature in Boston and in Chicago was also 7 degrees Celsius. Was the temperature in New York City the same as the temperature in Chicago? Explain your answer.

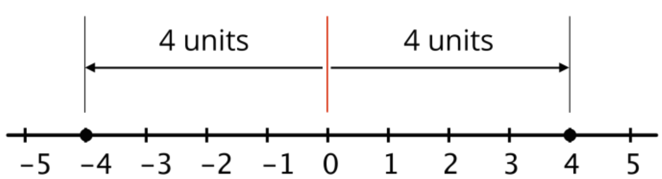
Lesson 6 Summary

We compare numbers by comparing their positions on the number line: the one farther to the right is greater; the one farther to the left is less.

Sometimes we wish to compare which one is closer to or farther from 0. For example, we may want to know how far away the temperature is from the freezing point of 0°C, regardless of whether it is above or below freezing.

The **absolute value** of a number tells us its distance from 0.

The absolute value of -4 is 4, because -4 is 4 units to the left of 0. The absolute value of 4 is also 4, because 4 is 4 units to the right of 0. Opposites always have the same absolute value because they both have the same distance from 0.



The distance from 0 to itself is 0, so the absolute value of 0 is 0. Zero is the *only* number whose distance to 0 is 0. For all other absolute values, there are always two numbers—one positive and one negative—that have that distance from 0.

To say “the absolute value of 4,” we write: $|4|$

To say that “the absolute value of -8 is 8,” we write: $|-8| = 8$

Lesson 6 Glossary Terms: absolute value

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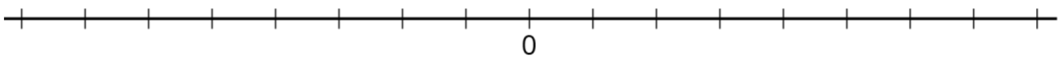
Unit 7, Lesson 7: Comparing Numbers and Distance from Zero

Let's use absolute value and negative numbers to think about elevation.

7.1: Opposites

1. a is a rational number. Choose a value for a and plot it on the number line.



2. a. Based on where you plotted a, plot $-a$ on the same number line.

   b. What is the value of $-a$ that you plotted?

3. Noah said, "If a is a rational number, $-a$ will always be a negative number." Do you agree with Noah? Explain your reasoning.

7.2: Submarine

A submarine is at an elevation of -100 feet (100 feet below sea level). Let's compare the elevations of these four people to that of the submarine:

- Clare's elevation is greater than the elevation of the submarine. Clare is farther from sea level than the submarine.
- Andre's elevation is less than the elevation of the submarine. Andre is farther away from sea level than the submarine.
- Han's elevation is greater than the elevation of the submarine. Han is closer to sea level than is the submarine.
- Lin's elevation is the same distance away from sea level as the submarine's.

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1. Complete the table as follows.
    a. Write a possible elevation for each person.
    b. Use <, >, or = to compare the elevation of that person to that of the submarine.
    c. Use absolute value to tell how far away the person is from sea level (elevation 0).

    As an example, the first row has been filled with a possible elevation for Clare.

	possible elevation	compare to submarine	distance from sea level
Clare	150 feet	150 > -100	|150| or 150 feet
Andre			
Han			
Lin			

2. Priya says her elevation is less than the submarine's and she is closer to sea level. Is this possible? Explain your reasoning.

7.3: Inequality Mix and Match

Here are some numbers and inequality symbols. Work with your partner to write true comparison statements.

-0.7	$-\frac{3}{5}$	1	4	$\|-8\|$	<
$-\frac{6}{3}$	-2.5	2.5	8	$\|0.7\|$	=
-4	0	$\frac{7}{2}$	$\|3\|$	$\|-\frac{5}{2}\|$	>

One partner should select two numbers and one comparison symbol and use them to write a true statement using symbols. The other partner should write a sentence in words with the same meaning, using the following phrases:

- is equal to
- is the absolute value of
- is greater than
- is less than

For example, one partner could write $4 < 8$ and the other would write, "4 is less than 8." Switch roles until each partner has three true mathematical statements and three sentences written down.

Write your mathematical statements and sentences in the space below.

Are you ready for more?

For each question, choose a value for each variable to make the whole statement true. (When the word *and* is used in math, both parts have to be true for the whole statement to be true.) Can you do it if one variable is negative and one is positive? Can you do it if both values are negative?

1. $x < y$ and $|x| < y$.
2. $a < b$ and $|a| < |b|$.
3. $c < d$ and $|c| > d$.
4. $t < u$ and $|t| > |u|$.

Lesson 7 Summary

We can use elevation to help us compare two rational numbers or two absolute values.

- Suppose an anchor has an elevation of -10 meters and a house has an elevation of 12 meters. To describe the anchor having a lower elevation than the house, we can write $-10 < 12$ and say "-10 is less than 12."
- The anchor is closer to sea level than the house is to sea level (or elevation of 0). To describe this, we can write $|-10| < |12|$ and say "the distance between -10 and 0 is less than the distance between 12 and 0."

We can use similar descriptions to compare rational numbers and their absolute values outside of the context of elevation.

- To compare the distance of -47.5 and 5.2 from 0, we can say: $|-47.5|$ is 47.5 units away from 0, and $|5.2|$ is 5.2 units away from 0, so $|-47.5| > |5.2|$.
- $|-18| > 4$ means that the absolute value of -18 is greater than 4. This is true because 18 is greater than 4.

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Unit 7, Lesson 8: Writing and Graphing Inequalities

Let's write inequalities.

8.1: Estimate Heights of People

1. Here is a picture of a man.

a. Name a number, in feet, that is clearly too high for this man's height.

b. Name a number, in feet, that is clearly too low for his height.

c. Make an estimate of his height.

Pause here for a class discussion.

2. Here is a picture of the same man standing next to a child.

If the man's actual height is 5 feet 10 inches, what can you say about the height of the child in this picture?

Be prepared to explain your reasoning.

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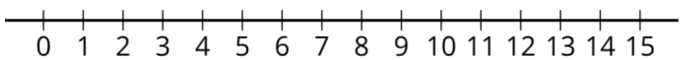
8.2: Stories about 9

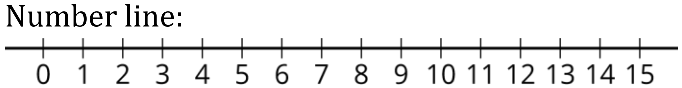
1. Your teacher will give you a set of paper slips with four stories and questions involving the number 9. Match each question to three representations of the solution: a description or a list, a number line, or an inequality statement.

2. Compare your matching decisions with another group's. If there are disagreements, discuss until both groups come to an agreement. Then, record your final matching decisions here.
   a. A fishing boat can hold fewer than 9 people. How many people (x) can it hold?
      - Description or list:
      - Number line:



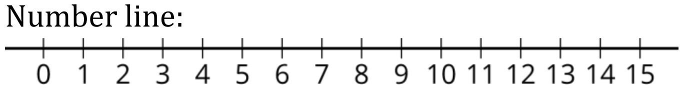
      - Inequality:

   b. Lin needs more than 9 ounces of butter to make cookies for her party. How many ounces of butter (x) would be enough?
      - Description or list:
      - Number line:



      - Inequality:

   c. A magician will perform her magic tricks only if there are at least 9 people in the audience. For how many people (x) will she perform her magic tricks?
      - Description or list:
      - Number line:

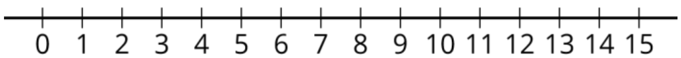


      - Inequality:

d. A food scale can measure up to 9 kilograms of weight. What weights (x) can the scale measure?

- Description or list:

- Number line:



- Inequality:

8.3: How High and How Low Can It Be?

Here is a picture of a person and a basketball hoop. Based on the picture, what do you think are reasonable estimates for the maximum and minimum heights of the basketball hoop?

1. Complete the first blank in each sentence with an estimate, and the second blank with "taller" or "shorter."

a. I estimate the *minimum* height of the basketball hoop to be ________ feet; this means the hoop cannot be ___________ than this height.

b. I estimate the *maximum* height of the basketball hoop to be ________ feet; this means the hoop cannot be ___________ than this height.

2. Write two inequalities—one to show your estimate for the *minimum* height of the basketball hoop, and another for the *maximum* height. Use an inequality symbol and the variable h to represent the unknown height.

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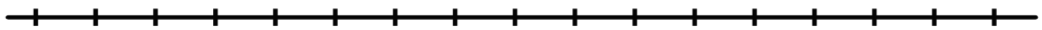
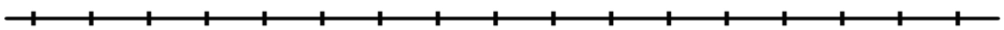
3. Plot each estimate for minimum or maximum value on a number line.

   Minimum:



   Maximum:

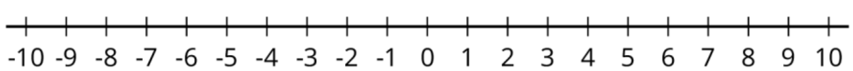


4. Suppose a classmate estimated the value of h to be 19 feet. Does this estimate agree with your inequality for the maximum height? Does it agree with your inequality for the minimum height? Explain or show how you know.

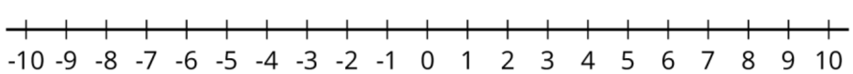
5. Ask a partner for an estimate of h. Record the estimate and check if it agrees with your inequalities for maximum and minimum heights.

Are you ready for more?

1. Find 3 different numbers that a could be if $|a| < 5$. Plot these points on the number line. Then plot as many other possibilities for a as you can.



2. Find 3 different numbers that b could be if $|b| > 3$. Plot these points on the number line. Then plot as many other possibilities for b as you can.



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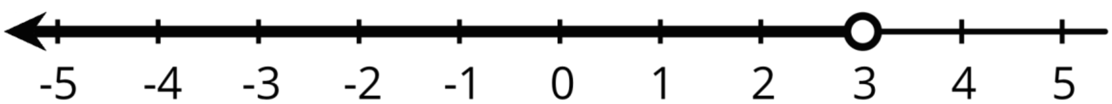
Lesson 8 Summary

An inequality tells us that one value is *less than* or *greater than* another value.

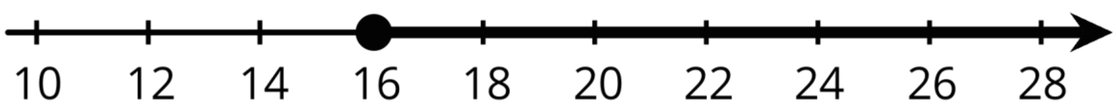
Suppose we knew the temperature is *less than* 3°F, but we don't know exactly what it is. To represent what we know about the temperature t in °F we can write the inequality: $t < 3$

The temperature can also be graphed on a number line. Any point to the left of 3 is a possible value for t. The open circle at 3 means that t cannot be *equal* to 3, because the temperature is *less than* 3.



Here is another example. Suppose a young traveler has to be at least 16 years old to fly on an airplane without an accompanying adult.

If a represents the age of the traveler, any number greater than 16 is a possible value for a, and 16 itself is also a possible value of a. We can show this on a number line by drawing a closed circle at 16 to show that it meets the requirement (a 16-year-old person can travel alone). From there, we draw a line that points to the right.



We can also write an inequality and equation to show possible values for a: $a > 16$ $a = 16$

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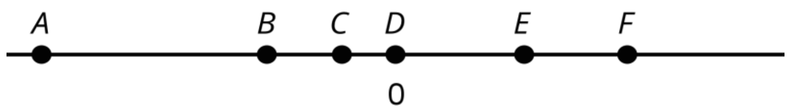
Unit 7, Lesson 9: Solutions of Inequalities

Let’s think about the solutions to inequalities.

9.1: Unknowns on a Number Line

The number line shows several points, each labeled with a letter.



1. Fill in each blank with a letter so that the inequality statements are true.

   a. ______ > ______ b. ______ < ______

2. Jada says that she found three different ways to complete the first question correctly. Do you think this is possible? Explain your reasoning.

3. List a possible value for each letter on the number line based on its location.

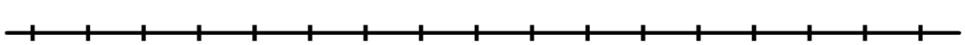
9.2: Amusement Park Rides

Priya finds these height requirements for some of the rides at an amusement park.

to ride the . . .	you must be . . .
High Bounce	between 55 and 72 inches tall
Climb-A-Thon	under 60 inches tall
Twirl-O-Coaster	58 inches minimum

1. Write an inequality for each of the the three height requirements. Use h for the unknown height. Then, represent each height requirement on a number line.

   High Bounce



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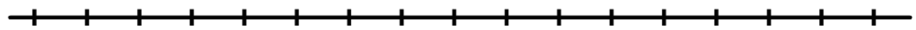
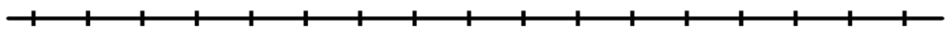
Climb-A-Thon



Twirl-O-Coaster



Pause here for additional instructions from your teacher.

2. Han's cousin is 55 inches tall. Han doesn't think she is tall enough to ride the High Bounce, but Kiran believes that she is tall enough. Do you agree with Han or Kiran? Be prepared to explain your reasoning.

3. Priya can ride the Climb-A-Thon, but she cannot ride the High Bounce or the Twirl-O-Coaster. Which, if any, of the following could be Priya's height? Be prepared to explain your reasoning.

   59 inches 53 inches 56 inches

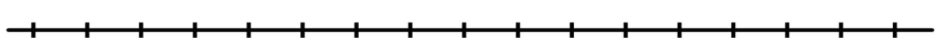
4. Jada is 56 inches tall. Which rides can she go on?

5. Kiran is 60 inches tall. Which rides can he go on?

6. The inequalities $h < 75$ and $h > 64$ represent the height restrictions, in inches, of another ride. Write three values that are **solutions** to both of these inequalities.

Are you ready for more?

1. Represent the height restrictions for all three rides on a single number line, using a different color for each ride.



2. Which part of the number line is shaded with all 3 colors?

3. Name one possible height a person could be in order to go on all three rides.

9.3: What Number Am I?

Your teacher will give your group two sets of cards—one set shows inequalities and the other shows numbers. Arrange the inequality cards face up where everyone can see them. Stack the number cards face down and shuffle them.

To play:

- Nominate one member of your group to be the detective. The other three players are clue givers.
- One clue giver picks a number from the stack and shows it only to the other clue givers. Each clue giver then chooses an inequality that will help the detective identify the unknown number.
- The detective studies the inequalities and makes three guesses.
- If the detective cannot guess the number correctly, the clue givers must choose an additional inequality to help. Add as many inequalities as needed to help the detective identify the correct number.
- When the detective succeeds, a different group member becomes the detective and everyone else is a clue giver.
- Repeat the game until everyone has had a turn playing the detective.

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Lesson 9 Summary

Let's say a movie ticket costs less than \$10. If c represents the cost of a movie ticket, we can use $c < 10$ to express what we know about the cost of a ticket.

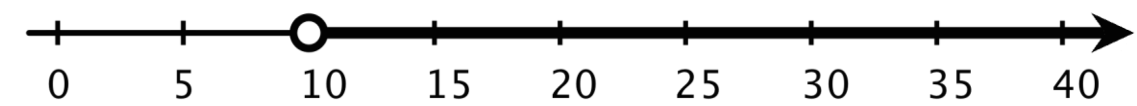
Any value of c that makes the inequality true is called a **solution to the inequality**.

For example, 5 is a solution to the inequality $c < 10$ because $5 < 10$ (or "5 is less than 10") is a true statement, but 12 is not a solution because $12 < 10$ ("12 is less than 10") is *not* a true statement.

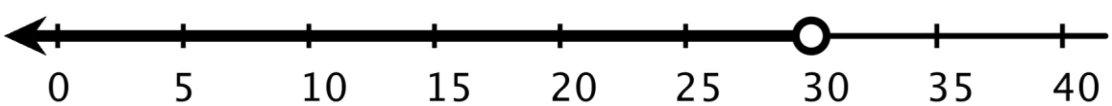
If a situation involves more than one boundary or limit, we will need more than one inequality to express it.

For example, if we knew that it rained for *more* than 10 minutes but *less* than 30 minutes, we can describe the number of minutes that it rained (r) with the following inequalities and number lines.

$r > 10$

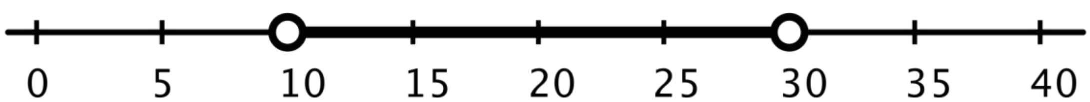


$r < 30$



Any number of minutes greater than 10 is a solution to $r > 10$, and any number less than 30 is a solution to $r < 30$. But to meet the condition of "more than 10 but less than 30," the solutions are limited to the numbers between 10 and 30 minutes, *not* including 10 and 30.

We can show the solutions visually by graphing the two inequalities on one number line.



Lesson 9 Glossary Terms

solution to an inequality

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Unit 7, Lesson 10: Interpreting Inequalities

Let's examine what inequalities can tell us.

10.1: True or False: Fractions and Decimals

Is each equation true or false? Be prepared to explain your reasoning.

1. $3(12 + 5) = (3 \cdot 12) \cdot (3 \cdot 5)$
2. $\frac{1}{3} \cdot \frac{3}{4} = \frac{3}{4} \cdot \frac{2}{6}$
3. $2 \cdot (1.5) \cdot 12 = 4 \cdot (0.75) \cdot 6$

10.2: Basketball Game

Noah scored n points in a basketball game.

1. What does $15 < n$ mean in the context of the basketball game?
2. What does $n < 25$ mean in the context of the basketball game?
3. Draw two number lines to represent the solutions to the two inequalities.
4. Name a possible value for n that is a solution to both inequalities.
5. Name a possible value for n that is a solution to $15 < n$, but not a solution to $n < 25$.
6. Can -8 be a solution to n in this context? Explain your reasoning.

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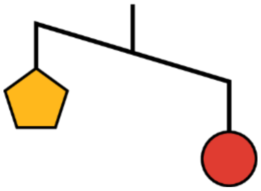
10.3: Unbalanced Hangers

1. Here is a diagram of an unbalanced hanger.

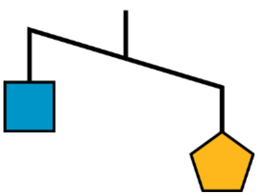


Jada says that the weight of one circle is greater than the weight of one pentagon.

   a. Write an inequality to represent her statement. Let p be the weight of one pentagon and c be the weight of one circle.

   b. A circle weighs 12 ounces. Use this information to write another inequality to represent the relationship of the weights. Then, describe what this inequality means in this context.

2. Here is another diagram of an unbalanced hanger.



   a. Write an inequality to represent the relationship of the weights. Let p be the weight of one pentagon and s be the weight of one square.

   b. One pentagon weighs 8 ounces. Use this information to write another inequality to represent the relationship of the weights. Then, describe what this inequality means in this context.

   c. Graph the solutions to this inequality on a number line.

3. Based on your work so far, can you tell the relationship between the weight of a square and the weight of a circle? If so, write an inequality to represent that relationship. If not, explain your reasoning.

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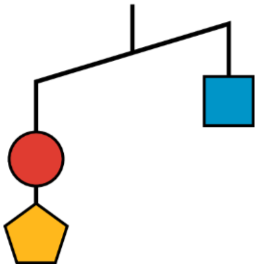
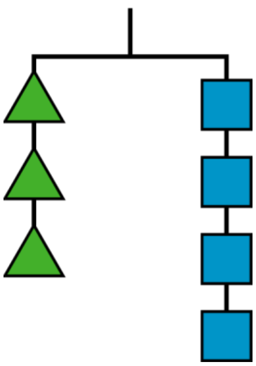
4. This is another diagram of an unbalanced hanger.



Andre writes the following inequality: $c + p < s$. Do you agree with his inequality? Explain your reasoning.

5. Jada looks at another diagram of an unbalanced hangar and writes: $s + c > 2t$, where t represents the weight of one triangle. Draw a sketch of the diagram.

Are you ready for more?



Here is a picture of a balanced hanger. It shows that the total weight of the three triangles is the same as the total weight of the four squares.

1. What does this tell you about the weight of one square when compared to one triangle? Explain how you know.

2. Write an equation or an inequality to describe the relationship between the weight of a square and that of a triangle. Let s be the weight of a square and t be the weight of a triangle.

Lesson 10 Summary

When we find the solutions to an inequality, we should think about its context carefully. A number may be a solution to an inequality outside of a context, but may not make sense when considered in context.

- Suppose a basketball player scored more than 11 points in a game, and we represent the number of points she scored, s, with the inequality $s > 11$. By looking only at $s > 11$, we can say that numbers such as 12, $14\frac{1}{2}$, and 130.25 are all solutions to the inequality because they each make the inequality true.

  $12 > 11$ $\quad$ $14\frac{1}{2} > 11$ $\quad$ $130.25 > 11$

  In a basketball game, however, it is only possible to score a whole number of points, so fractional and decimal scores are not possible. It is also highly unlikely that one person would score more than 130 points in a single game.

  In other words, the context of an inequality may limit its solutions.

Here is another example:

- The solutions to $r < 30$ can include numbers such as $27\frac{3}{4}$, 18.5, 0, and -7. But if r represents the number of minutes of rain yesterday (and it did rain), then our solutions are limited to positive numbers. Zero or negative number of minutes would not make sense in this context.

  To show the upper and lower boundaries, we can write two inequalities:

  $0 < r$ $\quad$ $r < 30$

Inequalities can also represent comparison of two unknown numbers.

- Let's say we knew that a puppy weighs more than a kitten, but we did not know the weight of either animal. We can represent the weight of the puppy, in pounds, with p and the weight of the kitten, in pounds, with k, and write this inequality:

$$p > k$$

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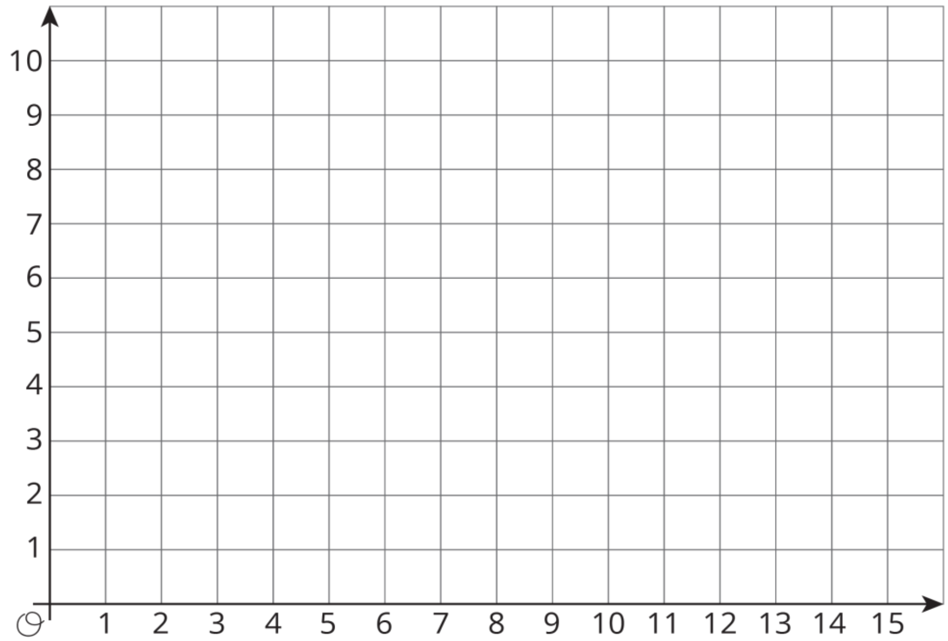
Unit 7, Lesson 11: Points on the Coordinate Plane

Let's explore and extend the coordinate plane.

11.1: Guess My Line



1. Choose a horizontal or a vertical line on the grid. Draw 4 points on the line and label each point with its coordinates.

2. Tell your partner whether your line is horizontal or vertical, and have your partner guess the locations of your points by naming coordinates.

   If a guess is correct, put an X through the point. If your partner guessed a point that is on your line but not the point that you plotted, say, "That point is on my line, but is not one of my points."

   Take turns guessing each other's points, 3 guesses per turn.

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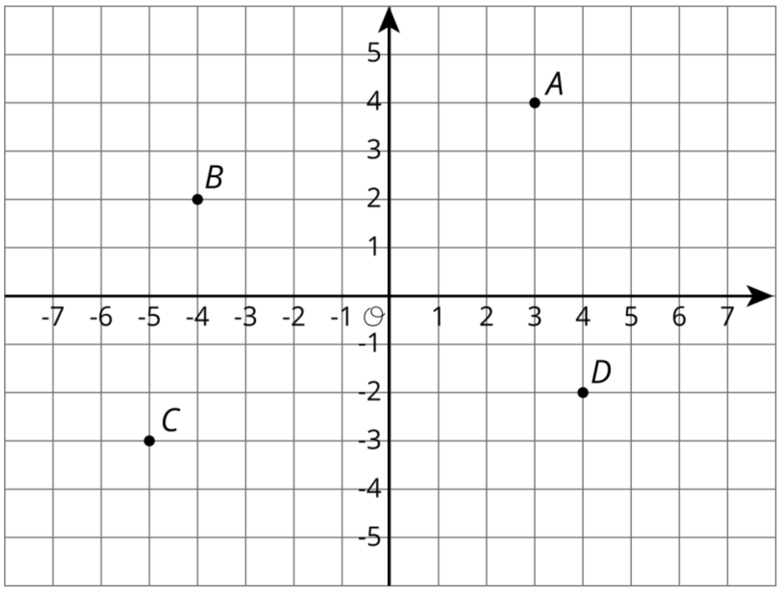
11.2: The Coordinate Plane

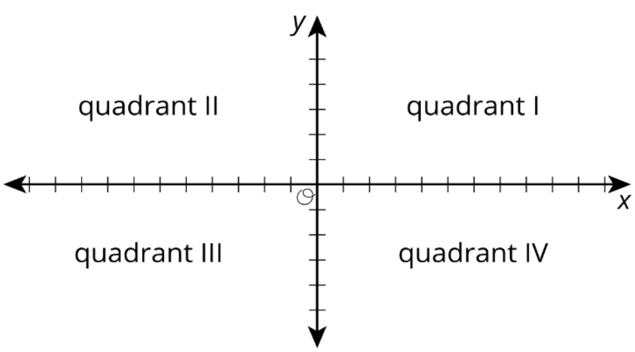


1. Label each point on the coordinate plane with an ordered pair.

2. What do you notice about the locations and ordered pairs of B, C, and D? How are they different from those for point A?

3. Plot a point at $(-2,5)$. Label it E. Plot another point at $(3,-4.5)$. Label it F.

4. The coordinate plane is divided into four **quadrants**, I, II, III, and IV, as shown here.



$G = (5,2)$

$H = (-1,-5)$

$I = (7,-4)$

   a. In which quadrant is G located? H? I?

   b. A point has a positive y-coordinate. In which quadrant could it be?

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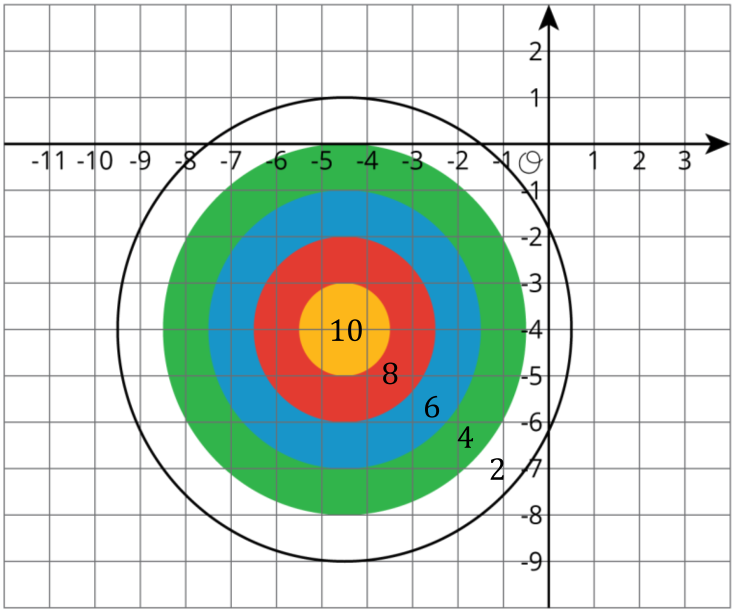
11.3: Coordinated Archery

Here is an image of an archery target on a coordinate plane. The scores for landing an arrow in the colored regions are shown.



Yellow (center): 10 points

Red: 8 points

Blue: 6 points

Green: 4 points

White (outer region): 2 points

Name the coordinates for a possible landing point to score.

1. 6 points
2. 10 points
3. 2 points
4. No points
5. 4 points
6. 8 points

Are you ready for more?

Pretend you are stuck in a coordinate plane. You can only take vertical and horizontal steps that are one unit long.

1. How many ways are there to get from the point $(-3,2)$ to $(-1,-1)$ if you will only step down and to the right?
2. How many ways are there to get from the point $(-1,-2)$ to $(4,0)$ if you can only step up and to the right?
3. Make up some more problems like this and see what patterns you notice.

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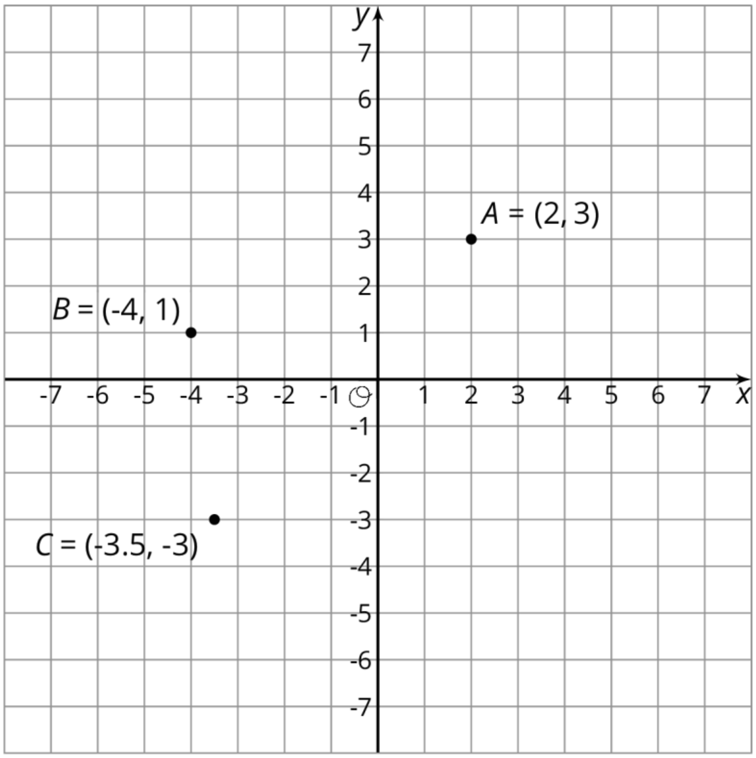
Lesson 11 Summary

Just as the number line can be extended to the left to include negative numbers, the x- and y-axis of a coordinate plane can also be extended to include negative values.



The ordered pair (x, y) can have negative x- and y-values. For $B = (-4,1)$, the x-value of -4 tells us that the point is 4 units to the left of the y-axis. The y-value of 1 tells us that the point is one unit above the x-axis.

The same reasoning applies to the points A and C. The x- and y-coordinates for point A are positive, so A is to the right of the y-axis and above the x-axis. The x- and y-coordinates for point C are negative, so C is to the left of the y-axis and below the x-axis.

Lesson 11 Glossary Terms

quadrant

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Unit 7, Lesson 12: Constructing the Coordinate Plane

Let's investigate different ways of creating a coordinate plane.

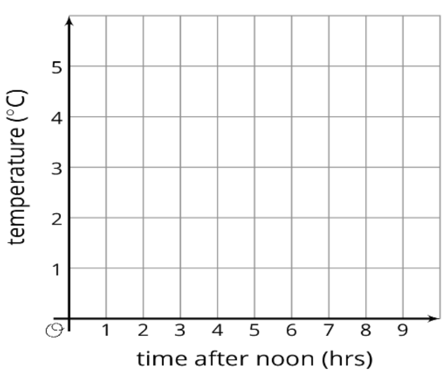
12.1: English Winter

The following data were collected over one December afternoon in England.

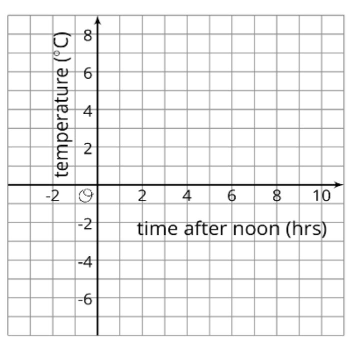
time after noon (hours)	temperature (°C)
0	5
1	3
2	4
3	2
4	1
5	-2
6	-3
7	-4
8	-4

1. Which set of axes would you choose to represent these data? Explain your reasoning.

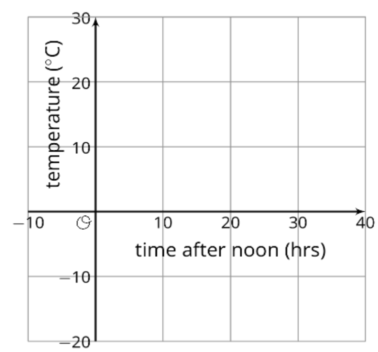
A



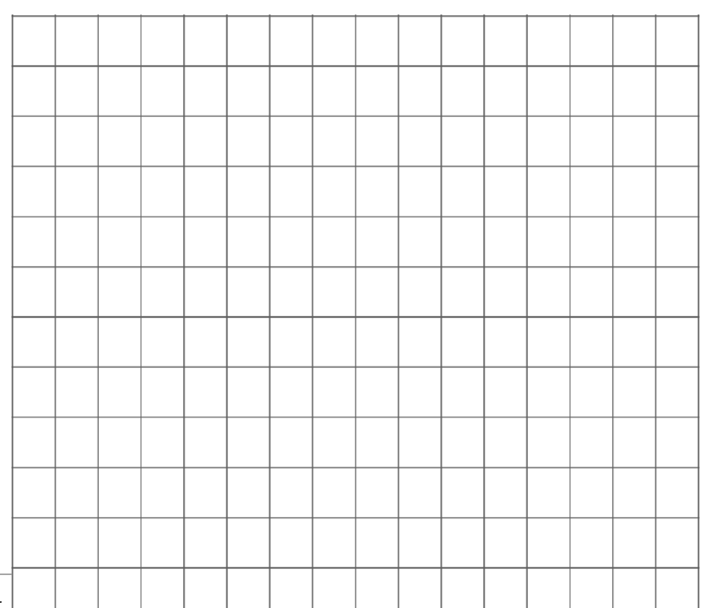
B



C



2. Explain why the other two sets of axes did not seem as appropriate as the one you chose.



12.2: Axes Drawing Decisions

1. Here are three sets of coordinates. For each set, draw and label an appropriate pair of axes and plot the points.

   a. $(1,2), (3,-4), (-5,-2), (0,2.5)$

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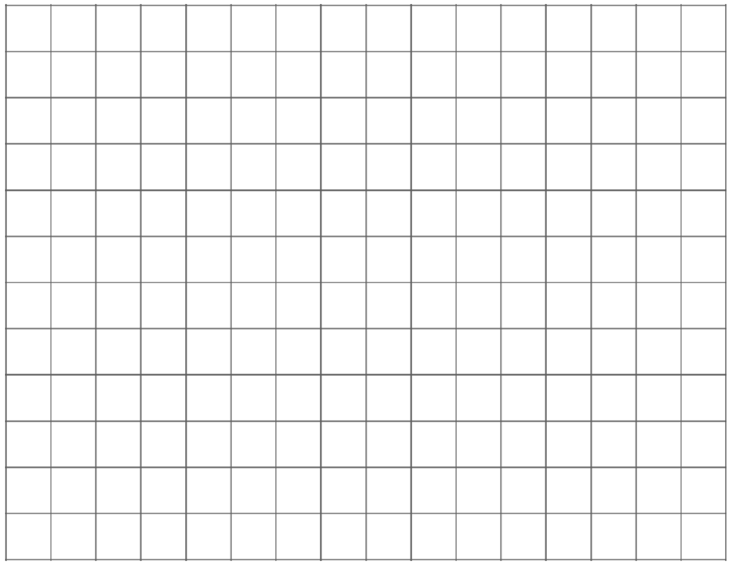
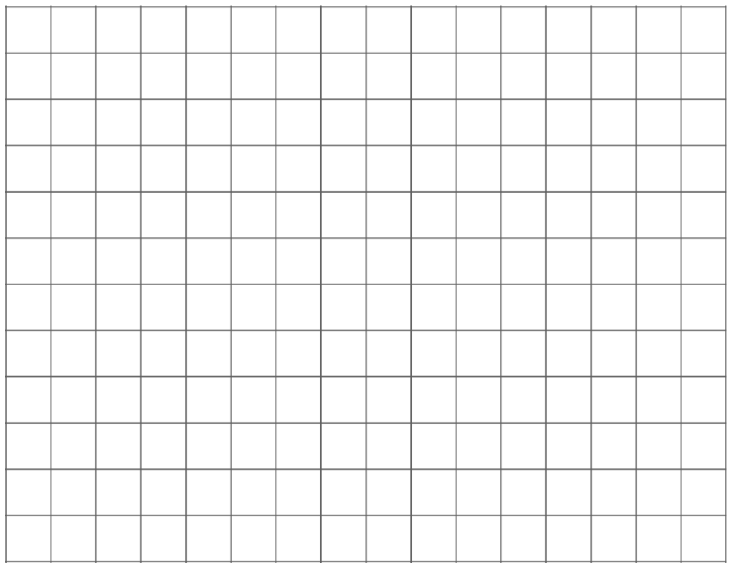
b. $(50,50), (0,0), (-10,-30), (-35,40)$



c. $\left(\frac{1}{4},\frac{3}{4}\right), \left(\frac{-5}{4},\frac{1}{2}\right), \left(-1\frac{1}{4},\frac{-3}{4}\right), \left(\frac{1}{4},\frac{-1}{2}\right)$



2. Discuss with a partner:

   - How are the axes and labels of your three drawings different?

   - How did the coordinates affect the way you drew the axes and label the numbers?

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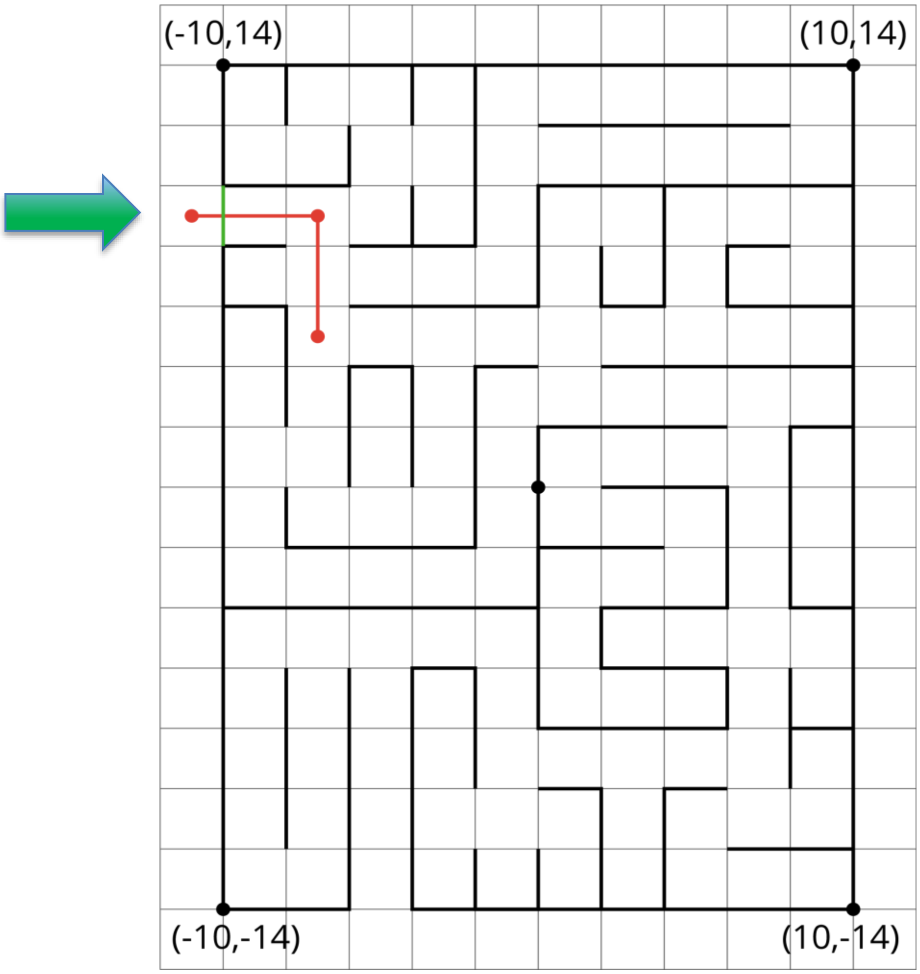
12.3: Positively A-maze-ing

Here is a maze on a coordinate plane. The black point in the center is (0, 0). The side of each grid square is 2 units long.



1. Enter the above maze at the location marked with a green segment (and the arrow). Draw line segments to show your way through and out of the maze. Label each turning point with a letter. Then, list all the letters and write their coordinates.

2. Choose any 2 turning points that share the same line segment. What is the same about their coordinates? Explain why they share that feature.

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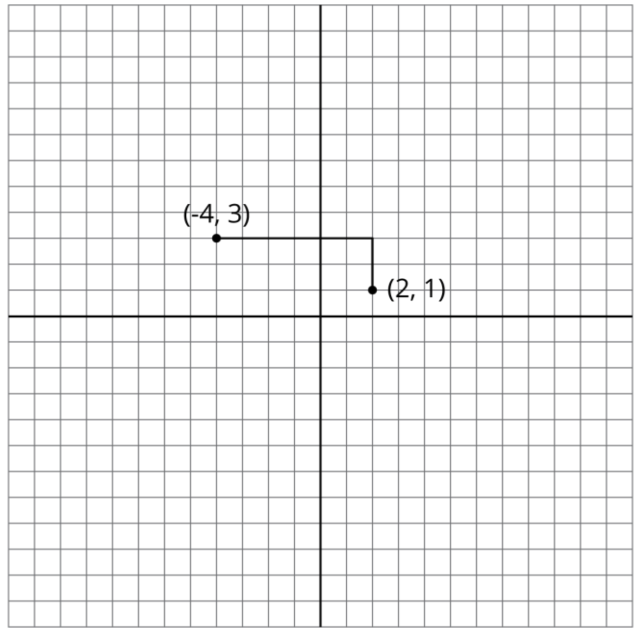
Are you ready for more?

To get from the point $(2,1)$ to $(-4,3)$ you can go two units up and six units to the left, for a total distance of eight units. This is called the "taxicab distance," because a taxi driver would have to drive eight blocks to get between those two points on a map.



Find as many points as you can that have a taxicab distance of eight units away from $(2,1)$. What shape do these points make?

Lesson 12 Summary

The coordinate plane can be used to show information involving pairs of numbers.

When using the coordinate plane, we should pay close attention to what each axis represents and what scale each uses.

Suppose we want to plot the following data about the temperatures in Minneapolis one evening.

time (hours from midnight)	temperature (degrees C)
-4	3
-1	-2
0	-4
3	-8

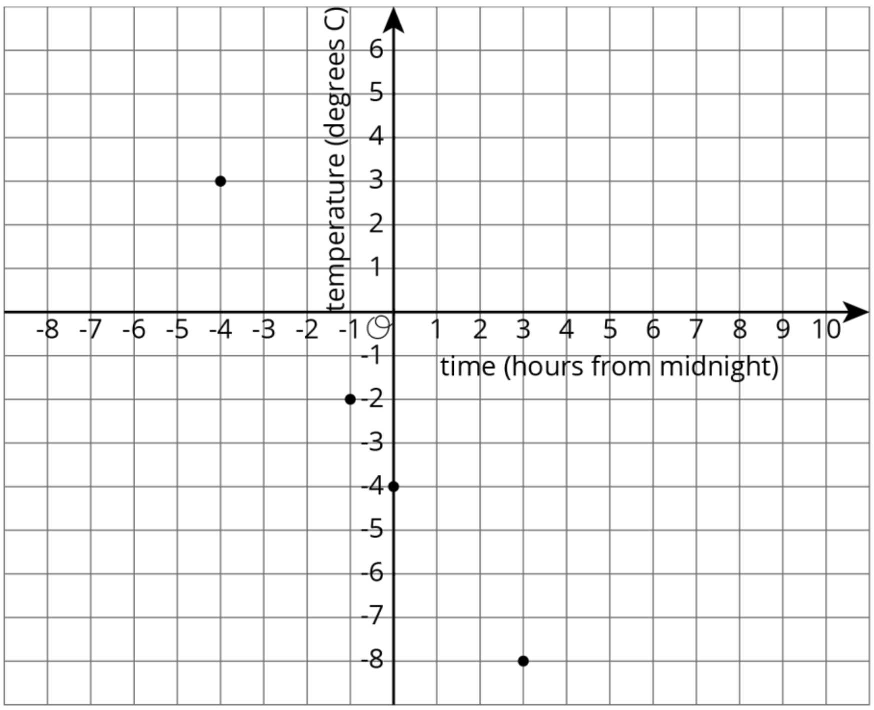
We can decide that the x-axis represents number of hours in relation to midnight and the y-axis represents temperatures in degrees Celsius.

- In this case, x-values less than 0 represent hours before midnight, and and x-values greater than 0 represent hours after midnight.
- On the y-axis, the values represents temperatures above and below the the freezing point of 0 degrees Celsius.

The data involve whole numbers, so it is appropriate that the each square on the grid represents a whole number.

- On the left of the origin, the x-axis needs to go as far as -4 or less (farther to the left). On the right, it needs to go to 3 or greater.
- Below the origin, the y-axis has to go as far as -8 or lower. Above the origin, it needs to go to 3 or higher.

Here is a graph of the data with the axes labeled appropriately.



On this coordinate plane, the point at (0,0) means a temperature of 0 degrees Celsius at midnight. The point at $(-2,8)$ means a temperature of 8 degree Celsius at 2 hours before midnight (or 10 p.m.).

Unit 7, Lesson 13: Interpreting Points on a Coordinate Plane

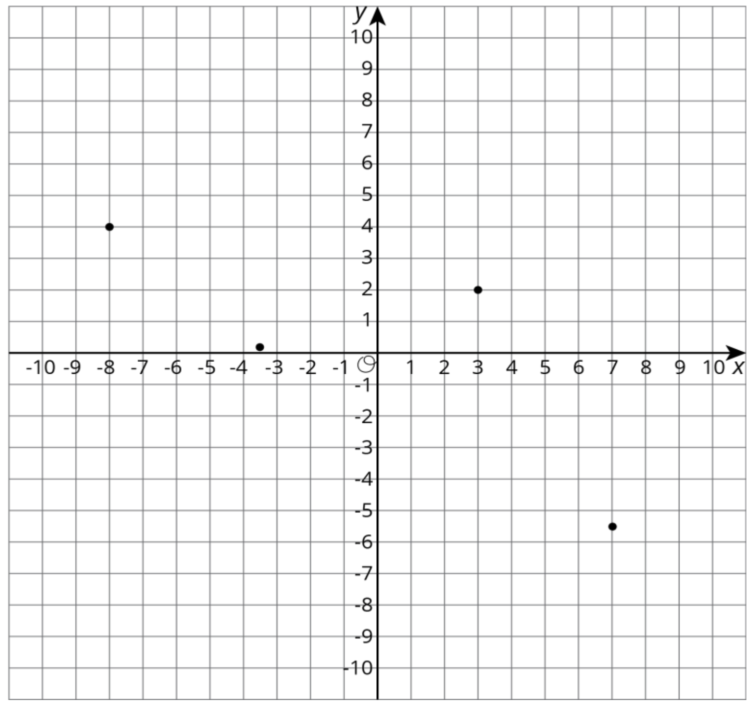
Let's examine what points on the coordinate plane can tell us.

13.1: Unlabeled Points

Label each point on the coordinate plane with the appropriate letter and ordered pair.

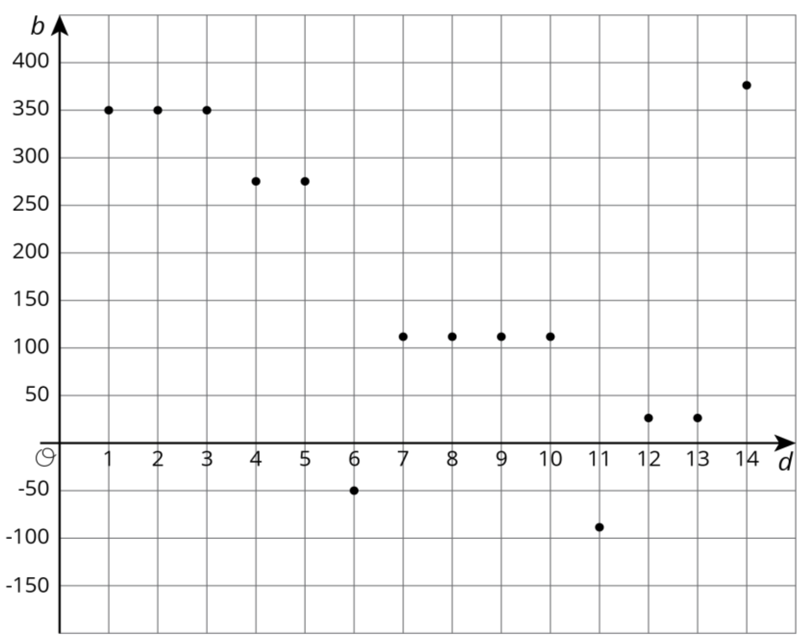
$A = (7, -5.5)$ $B = (-8, 4)$

$C = (3, 2)$ $D = (-3.5, 0.2)$



13.2: Account Balance

The graph shows the balance in a bank account over a period of 14 days. The axis labeled b represents account balance in dollars. The axis labeled d represents the day.



1. Estimate the greatest account balance. On which day did it occur?

2. Estimate the least account balance. On which day did it occur?

3. What does the point $(6, -50)$ tell you about the account balance?

4. How can we interpret $|-50|$ in the context?

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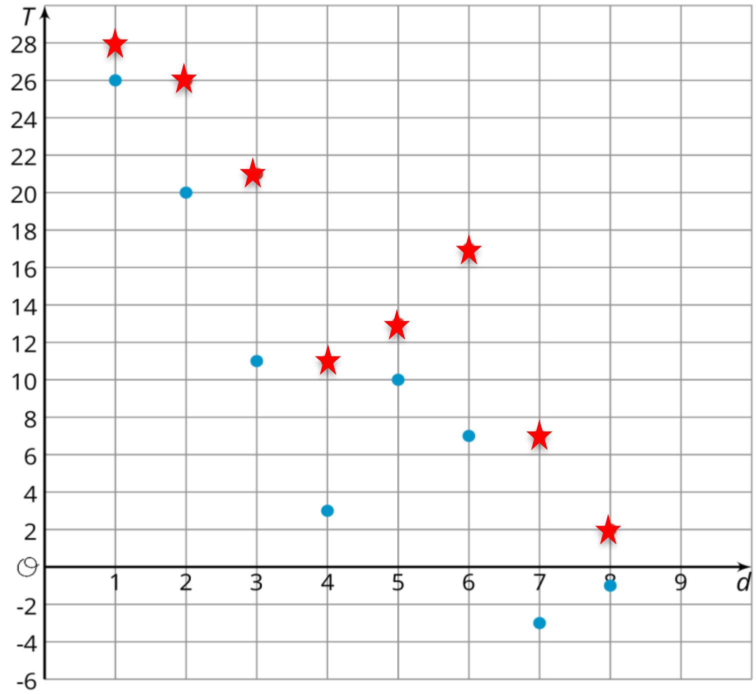
13.3: High and Low Temperatures

The coordinate plane shows the high (red stars) and low temperatures (blue dots) in Nome, Alaska over a period of 8 days. The axis labeled T represents temperatures in degrees Fahrenheit. The axis labeled d represents the day.



1. a. What was the warmest high temperature?

   b. Write an inequality to describe the high temperatures, H, over the 8-day period.

2. a. What was the coldest low temperature?

   b. Write an inequality to describe the low temperatures, L, over the 8-day period.

3. a. On which day(s) did the *largest* difference between the high and low temperatures occur? Write down this difference.

   b. On which day(s) did the *smallest* difference between the high and low temperatures occur? Write down this difference.

Are you ready for more?

Before doing this problem, do the problem about taxicab distance in an earlier lesson.

The point $(0,4)$ is 4 taxicab units away from $(-4,3)$ and 4 taxicab units away from $(2,1)$.

1. Find as many other points as you can that are 4 taxicab units away from *both* $(-4,3)$ and $(2,1)$.

2. Are there any points that are 3 taxicab units away from both points?

Lesson 13 Summary

Points on the coordinate plane can give us information about a context or a situation. One of those contexts is about money.

To open a bank account, we have to put money into the account. The account balance is the amount of money in the account at any given time. If we put in $350 when opening the account, then the account balance will be 350.

Sometimes we may have no money in the account and need to borrow money from the bank. In that situation, the account balance would have a negative value. If we borrow $200, then the account balance is -200.

A coordinate grid can be used to display both the balance and the day or time for any balance. This allows to see how the balance changes over time or to compare the balances of different days.

Similarly, if we plot on the coordinate plane data such as temperature over time, we can see how temperature changes over time or compare temperatures of different times.

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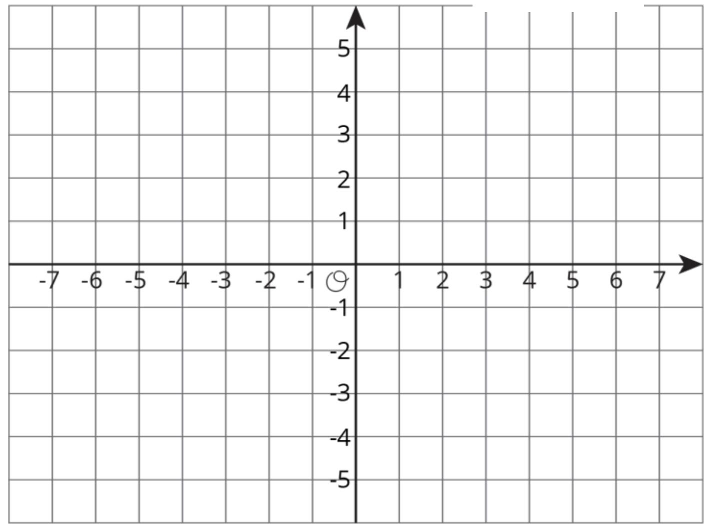
Unit 7, Lesson 14: Distances on a Coordinate Plane

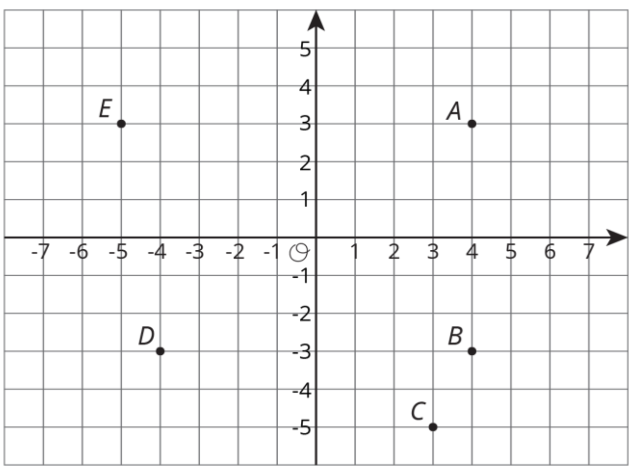
Let's explore distance on the coordinate plane.

14.1: Coordinate Patterns

Plot points in your assigned quadrant and label them with their coordinates.



14.2: Signs of Numbers in Coordinates



1. Write the coordinates of each point.

$A =$

$B =$

$C =$

$D =$

$E =$

2. Answer these questions for each pair of points.

   - How are the coordinates the same? How are they different?
   - How far away are they from the y-axis? To the left or to the right of it?
   - How far away are they from the x-axis? Above or below it?

   a. *A* and *B* 2. *B* and *D* 3. *A* and *D*

Pause here for a class discussion.

3. Point F has the same coordinates as point C, except its y-coordinate has the opposite sign.
   a. Plot point F on the coordinate plane and label it with its coordinates.
   b. How far away are F and C from the x-axis?
   c. What is the distance between F and C?

4. Point G has the same coordinates as point E, except its x-coordinate has the opposite sign.
   a. Plot point G on the coordinate plane and label it with its coordinates.
   b. How far away are G and E from the y-axis?
   c. What is the distance between G and E?

5. Point H has the same coordinates as point B, except its *both* coordinates have the opposite sign. In which quadrant is point H?

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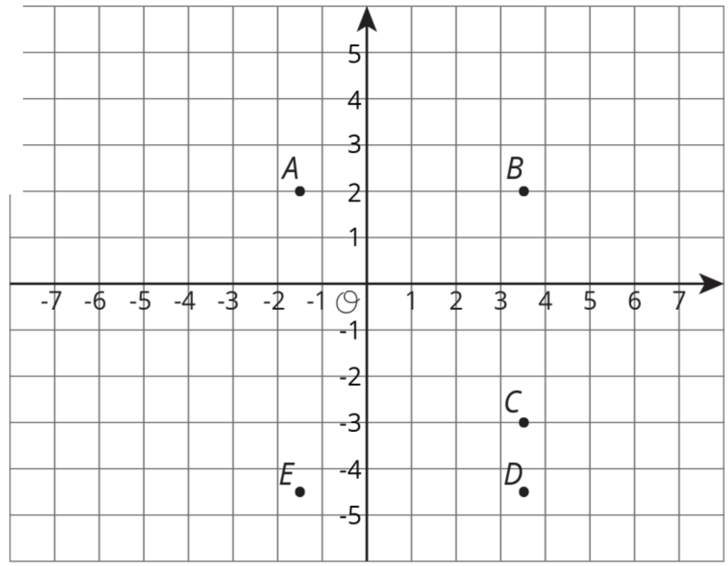
14.3: Finding Distances on a Coordinate Plane

1. Label each point with its coordinates.



2. Find the distance between each of the following pairs of points.

   a. Point B and C

   b. Point D and B

   c. Point D and E

3. Which of the points are 5 units from $(-1.5, -3)$?

4. Which of the points are 2 units from $(0.5, -4.5)$?

5. Plot a point that is both 2.5 units from A and 9 units from E. Label that point M and write down its coordinates.

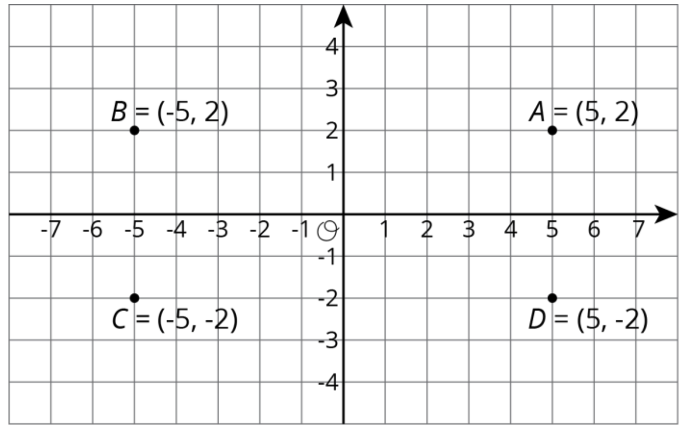
Are you ready for more?

Priya says, "There are exactly four points that are 3 units away from $(-5,0)$." Lin says, "I think there are a whole bunch of points that are 3 units away from $(-5,0)$."

Do you agree with either of them? Explain your reasoning.

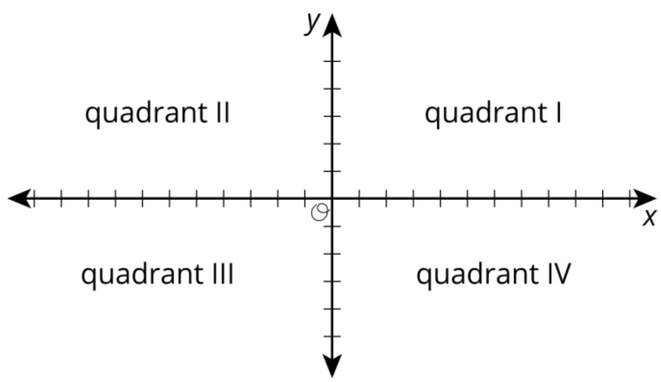
Lesson 14 Summary

The points $A = (5,2)$, $B = (-5,2)$, $C = (-5,-2)$, and $D = (5,-2)$ are shown in the plane. Notice that they all have almost the same coordinates, except the signs are different. They are all the same distance from each axis but are in different quadrants.



We can always tell which quadrant a point is located in by the signs of its coordinates.

x	y	quadrant
positive	positive	I
negative	positive	II
negative	negative	III
positive	negative	IV



In general:

- If two points have x-coordinates that are opposites (like 5 and -5), they are the same distance away from the vertical axis, but one is to the left and the other to the right.
- If two points have y-coordinates that are opposites (like 2 and -2), they are the same distance away from the horizontal axis, but one is above and the other below.

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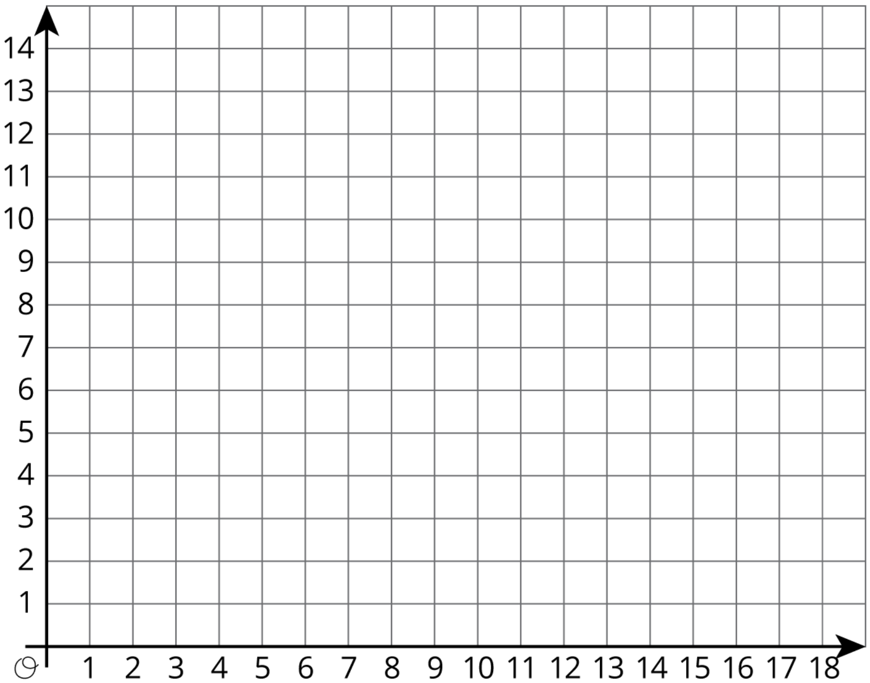
Unit 7, Lesson 15: Shapes on the Coordinate Plane

Let's use the coordinate plane to solve problems and puzzles.

15.1: Figuring Out The Coordinate Plane



1. Draw a figure in the coordinate plane with at least three of following properties:
    - 6 vertices
    - 1 pair of parallel sides
    - At least 1 right angle
    - 2 sides with the same length

2. Is your figure a polygon? Explain how you know.

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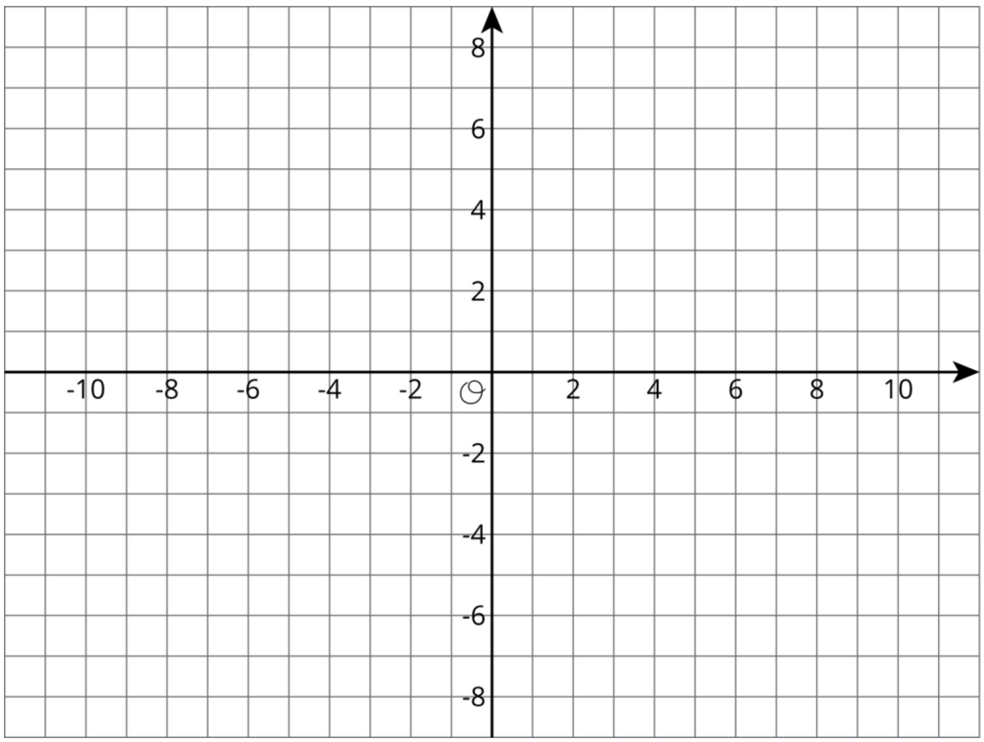
15.2: Plotting Polygons

Here are the coordinates for four polygons. Plot them on the coordinate plane, connect the points in the order that they are listed, and label each polygon with its letter name.

1. Polygon A: $(-7,4), (-8,5), (-8,6), (-7,7), (-5,7), (-5,5), (-7,4)$
2. Polygon B: $(4,3), (3,3), (2,2), (2,1), (3,0), (4,0), (5,1), (5,2), (4,3)$
3. Polygon C: $(-8,-5), (-8,-8), (-5,-8), (-5,-5), (-8,-5)$
4. Polygon D: $(-5,1), (-3,-3), (-1,-2), (0,3), (-3,3), (-5,1)$



Are you ready for more?

Find the area of Polygon D in this activity.

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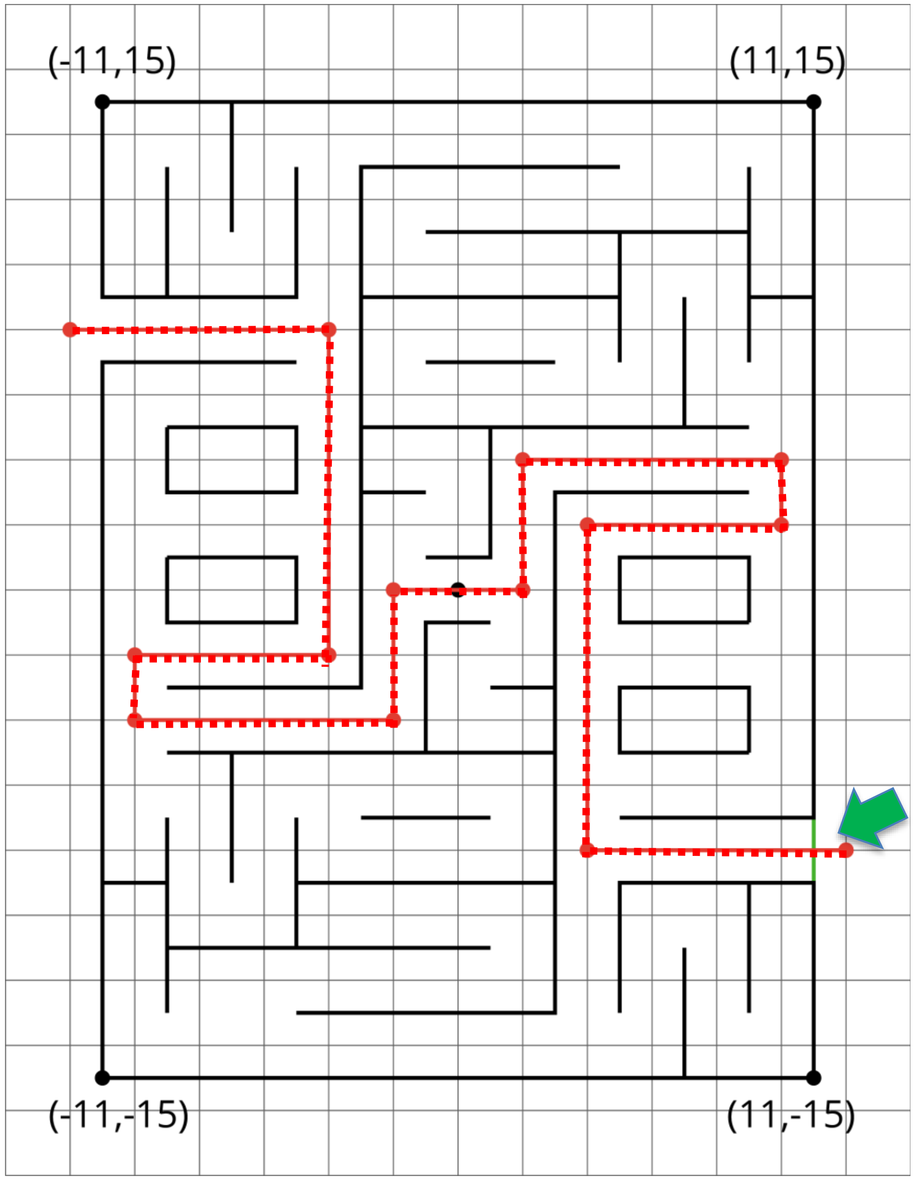
15.3: Four Quadrants of A-Maze-ing

1. The following diagram shows Andre's route through a maze. He started from the lower right entrance (see arrow).



   a. What are the coordinates of the first two and the last two points of his route?

   b. How far did he walk from his starting point to his ending point? Show how you know.

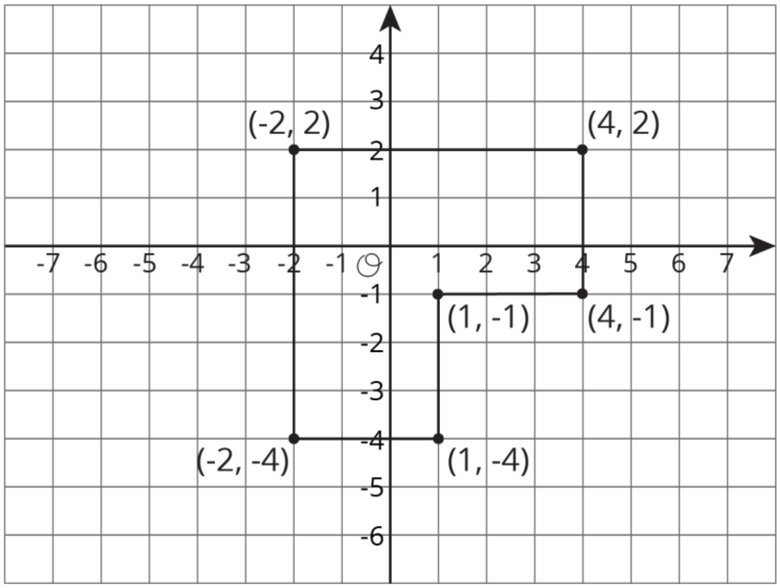
2. Jada went into the maze and stopped at $(-7,2)$.

   a. Plot that point and other points that would lead her out of the maze (through the exit on the upper left side).

   b. How far from $(-7,2)$ must she walk to exit the maze? Show how you know.

Lesson 15 Summary

We can use coordinates to find lengths of segments in the coordinate plane.



For example, we can find the perimeter of this polygon by finding the sum of its side lengths. Starting from $(-2,2)$ and moving clockwise, we can see that the lengths of the segments are 6, 3, 3, 3, 3, and 6 units. The perimeter is therefore 24 units.

In general:

- If two points have the same x-coordinate, they will be on the same vertical line, and we can find the distance between them.
- If two points have the same y-coordinate, they will be on the same horizontal line, and we can find the distance between them.

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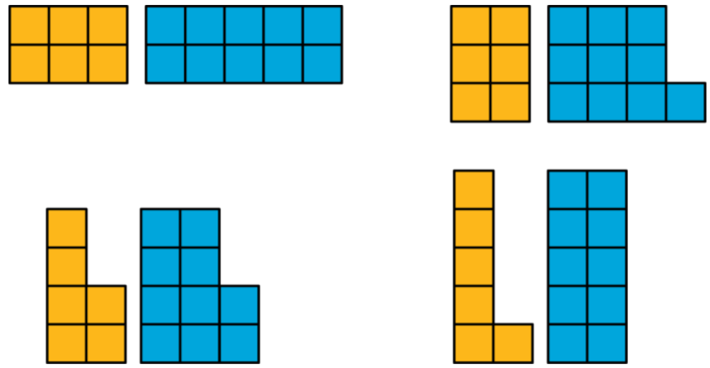
Unit 7, Lesson 16: Common Factors

Let’s use factors to solve problems.

16.1: Figures Made of Squares

How are the pairs of figures alike? How are they different?



16.2: Diego’s Bake Sale

Diego is preparing brownies and cookies for a bake sale. He would like to make equal-size bags for selling all of the 48 brownies and 64 cookies that he has. Organize your answer to each question so that it can be followed by others.

1. How can Diego package all the 48 brownies so that each bag has the same number of them? How many bags can he make, and how many brownies will be in each bag? Find all the possible ways to package the brownies.

2. How can Diego package all the 64 cookies so that each bag has the same number of them? How many bags can he make, and how many cookies will be in each bag? Find all the possible ways to package the cookies.

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3. How can Diego package all the 48 brownies and 64 cookies so that each bag has the same combination of items? How many bags can he make, and how many of each will be in each bag? Find all the possible ways to package both items.

4. What is the largest number of combination bags that Diego can make with no left over? Explain to your partner how you know that it is the largest possible number of bags.

16.3: Greatest Common Factor

1. The **greatest common factor** of 30 and 18 is 6. What do you think the term "greatest common factor" means?

2. Find all of the **factors** of 21 and 6. Then, identify the greatest common factor of 21 and 6.

3. Find all of the factors of 28 and 12. Then, identify the greatest common factor of 28 and 12.

4. A rectangular bulletin board is 12 inches tall and 27 inches wide. Elena plans to cover it with squares of colored paper that are all the same size. The paper squares come in different sizes; all of them have whole-number inches for their side lengths.

   a. What is the side length of the largest square that Elena could use to cover the bulletin board completely without gaps and overlaps? Explain or show your reasoning.

b. How is the solution to this problem related to greatest common factor?

Are you ready for more?

A school has 1000 lockers, all lined up in a hallway. Each locker is closed. Then...

- One student goes down the hall and opens each locker.
- A second student goes down the hall and closes every second locker: lockers 2, 4, 6, and so on.
- A third student goes down the hall and changes every third locker. If a locker is open, he closes it. If a locker is closed, he opens it.
- A fourth student goes down the hall and changes every fourth locker.

This process continues up to the thousandth student! At the end of the process, which lockers will be open? (Hint: you may want to try this problem with a smaller number of lockers first.)

Lesson 16 Summary

A factor of a whole number n is a whole number that divides n evenly without a remainder. For example, 1, 2, 3, 4, 6, and 12 are all factors of 12 because each of them divides 12 evenly and without a remainder.

A **common factor** of two whole numbers is a factor that they have in common. For example, 1, 3, 5, and 15 are factors of 45; they are also factors of 60. We call 1, 3, 5, and 15 common factors of 45 and 60.

The **greatest common factor** (sometimes written as GCF) of two whole numbers is the greatest of all of the common factors. For example, 15 is the greatest common factor for 45 and 60.

One way to find the greatest common factor of two whole numbers is to list all of the factors for each, and then look for the greatest factor they have in common. Let's try to find the greatest common factor of 18 and 24. First, we list all the factors of each number.

- Factors of 18: **1**, **2**, **3**, **6**, 9,18
- Factors of 24: **1**, **2**, **3**, 4, **6**, 8, 12, 24

The common factors are 1, 2, 3, and 6. Of these, 6 is the greatest one, so 6 is the greatest common factor of 18 and 24.

Lesson 16 Glossary Terms

common factor greatest common factor

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Unit 7, Lesson 17: Common Multiples

Let's use multiples to solve problems.

17.1: Notice and Wonder: Multiples

Circle all the multiples of 4 in this list.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

Circle all the multiples of 6 in this list.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

What do you notice? What do you wonder?

17.2: The Florist's Order

A florist can order roses in bunches of 12 and lilies in bunches of 8. Last month she ordered the same number of roses and lilies.

1. If she ordered no more than 100 of each kind of flower, how many bunches of each could she have ordered? Find all the possible combinations.

2. What is the smallest number of bunches of roses that she could have ordered? What about the smallest number of bunches of lilies? Explain your reasoning.

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17.3: Least Common Multiple

The **least common multiple** of 6 and 8 is 24.

1. What do you think the term “least common multiple” means?

2. Find all of the **multiples** of 10 and 8 that are less than 100. Find the least common multiple of 10 and 8.

3. Find all of the multiples of 7 and 9 that are less than 100. Find the least common multiple of 7 and 9.

Are you ready for more?

1. What is the least common multiple of 10 and 20?

2. What is the least common multiple of 4 and 24?

3. In the previous two questions, one number is a multiple of the other. What do you notice about their least common multiple? Do you think this will always happen when one number is a multiple of the other? Explain your reasoning.

17.4: Prizes on Grand Opening Day

Lin’s uncle is opening a bakery. On the bakery’s grand opening day, he plans to give away prizes to the first 50 customers that enter the shop. Every fifth customer will get a free bagel. Every ninth customer will get a free blueberry muffin. Every 12th customer will get a free slice of carrot cake.

1. Diego is waiting in line and is the 23rd customer. He thinks that he should get farther back in line in order to get a prize. Is he right? If so, how far back should he go to get at least one prize? Explain your reasoning.

2. Jada is the 36th customer.

   a. Will she get a prize? If so, what prize will she get?

   b. Is it possible for her to get more than one prize? How do you know? Explain your reasoning.

3. How many prizes total will Lin's uncle give away? Explain your reasoning.

Lesson 17 Summary

A multiple of a whole number is a product of that number with another whole number. For example, 20 is a multiple of 4 because $20 = 5 \cdot 4$.

A **common multiple** for two whole numbers is a number that is a multiple of both numbers. For example, 20 is a multiple of 2 and a multiple of 5, so 20 is a common multiple of 2 and 5.

The **least common multiple** (sometimes written as LCM) of two whole numbers is the smallest multiple they have in common. For example, 30 is the least common multiple of 6 and 10.

One way to find the least common multiple of two numbers is to list multiples of each in order until we find the smallest multiple they have in common. Let's find the least common multiple for 4 and 10. First, we list some multiples of each number.

- Multiples of 4: 4, 8, 12, 16, **20**, 24, 28, 32, 36, **40**, 44...
- Multiples of 10: 10, **20**, 30, **40**, 50, ...

20 and 40 are both common multiples of 4 and 10 (as are 60, 80, . . .), but 20 is the smallest number that is on *both* lists, so 20 is the least common multiple.

Lesson 17 Glossary Terms

least common multiple
common multiple

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Unit 7, Lesson 18: Using Common Multiples and Common Factors

Let's use common factors and common multiple to solve problems.

18.1: Keeping a Steady Beat

Your teacher will give you instructions for playing a rhythm game. As you play the game, think about these questions:

- When will the two sounds happen at the same time?
- How does this game relate to common factors or common multiples?

18.2: Factors and Multiples

Work with your partner to solve the following problems.

1. **Party.** Elena is buying cups and plates for her party. Cups are sold in packs of 8 and plates are sold in packs of 6. She wants to have the same number of plates and cups.
    a. Find a number of plates and cups that meets her requirement.
    b. How many packs of each supply will she need to buy to get that number?
    c. Name two other quantities of plates and cups she could get to meet her requirement.
2. **Tiles**. A restaurant owner is replacing the restaurant's bathroom floor with square tiles. The tiles will be laid side-by-side to cover the entire bathroom with no gaps, and none of the tiles can be cut. The floor is a rectangle that measures 24 feet by 18 feet.
    a. What is the largest possible tile size she could use? Write the side length in feet. Explain how you know it's the largest possible tile.
    b. How many of these largest size tiles are needed?

c. Name more tile sizes that are whole number of feet that she could use to cover the bathroom floor. Write the side lengths (in feet) of the square tiles.

3. **Stickers**. To celebrate the first day of spring, Lin is putting stickers on some of the 100 lockers along one side of her middle school's hallway. She puts a skateboard sticker on every 4th locker (starting with locker 4), and a kite sticker on every 5th locker (starting with locker 5).

   a. Name three lockers that will get both stickers.

   b. After Lin makes her way down the hall, will the 30th locker have no stickers, 1 sticker, or 2 stickers? Explain how you know.

4. **Kits.** The school nurse is assembling first-aid kits for the teachers. She has 75 bandages and 90 throat lozenges. All the kits must have the same number of each supply, and all supplies must be used.

   a. What is the largest number of kits the nurse can make?

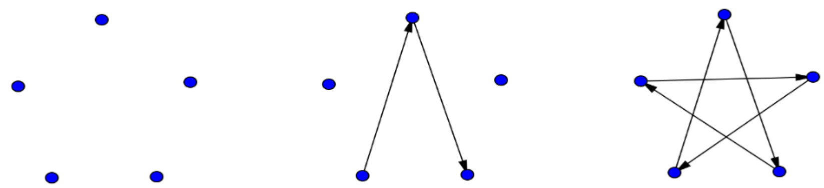
   b. How many bandages and lozenges will be in each kit?

5. What kind of mathematical work was involved in each of the previous problems? Put a checkmark to show what the questions were about.

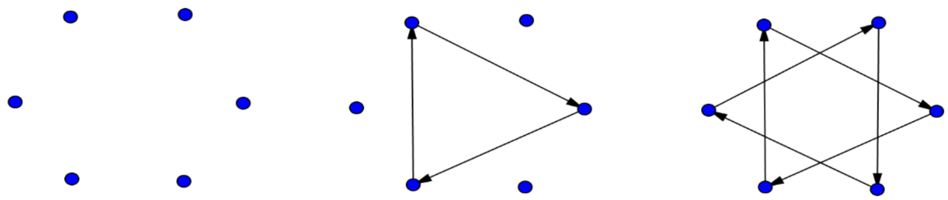
problem	finding multiples	finding least common multiple	finding factors	finding greatest common factor
Party				
Tiles				
Stickers				
Kits				

Are you ready for more?

You probably know how to draw a five-pointed star without lifting your pencil. One way to do this is to start with five dots arranged in a circle, then connect every second dot.

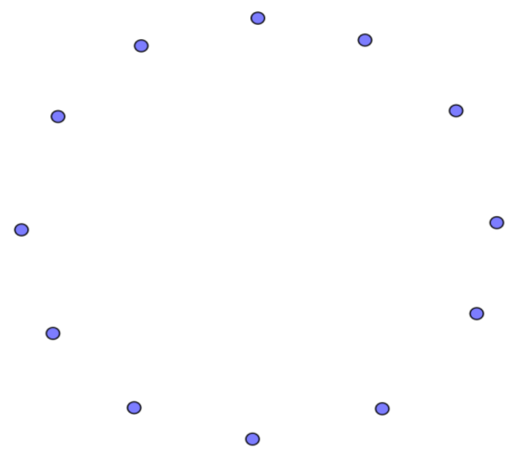


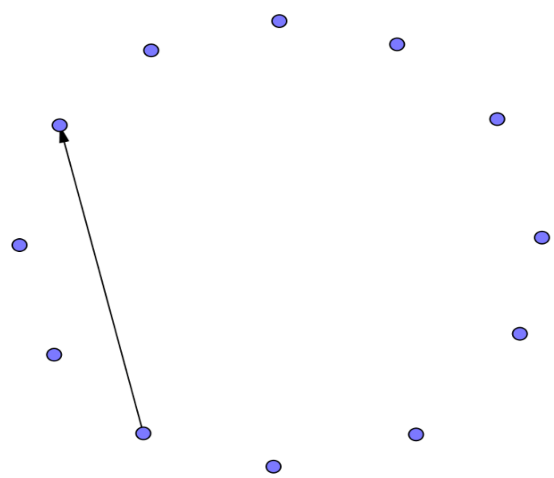
If you try the same thing with six dots arranged in a circle, you will have to lift your pencil. Once you make the first triangle, you'll have to find an empty dot and start the process over. Your six-pointed star has two pieces that are each drawn without lifting the pencil.



With twelve dots arranged in a circle, we can make some twelve-pointed stars.

1. Start with one dot and connect every second dot, as if you were drawing a five-pointed star. Can you draw the twelve-pointed star without lifting your pencil? If not, how many pieces does the twelve-pointed star have?





2. This time, connect every third dot. Can you draw this twelve-pointed star without lifting your pencil? If not, how many pieces do you get?

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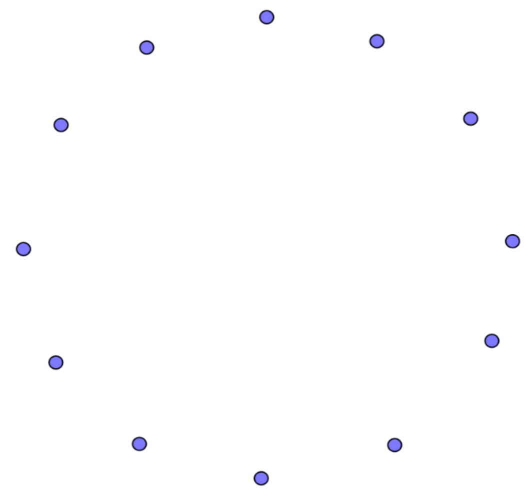
3. What do you think will happen if you connect every fourth dot? Try it. How many pieces do you get?

4. Do you think there is any way to draw a twelve-pointed star without lifting your pencil? Try it out.



5. Now investigate eight-pointed stars, nine-pointed stars, and ten-pointed stars. What patterns do you notice?

18.3: More Factors and Multiples

Here are five more problems. Read and discuss each one with your group. *Without solving*, predict whether each problem involves finding common multiples or finding common factors. Circle one or more options to show your prediction.

1. **Soccer.** Diego and Andre are both in a summer soccer league. During the month of August, Diego has a game every 3rd day, starting August 3rd, and Andre has a game every 4th day, starting August 4th.

   - common multiples
   - least common multiple
   - common factors
   - greatest common factor

   a. What is the first date that both boys will have a game?

   b. How many of their games fall on the same date?

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2. **Performances.** During a performing arts festival, students from elementary and middle schools will be grouped together for various performances. There are 32 elementary students and 40 middle-school students. The arts director wants identical groups for the performances, with students from both schools in each group. Each student can be a part of only one group.

- ○ common multiples
- ○ least common multiple
- ○ common factors
- ○ greatest common factor

a. Name all possible groupings.

b. What is the largest number of groups that can be formed? How many elementary school students and how many middle school students will be in each group?

3. **Lights.** There is a string of holiday lights with red, gold, and blue lights. The red lights are set to blink every 12 seconds, the gold lights are set to blink every 8 seconds, and the blue lights are set to blink every 6 seconds. The lights are on an automatic timer that starts each day at 7:00 p.m. and stops at midnight.

- ○ common multiples
- ○ least common multiple
- ○ common factors
- ○ greatest common factor

a. After how much time with all 3 lights blink at the exact same time?

b. How many times total will this happen in one day?

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4. **Banners.** Noah has two pieces of cloth. He is making square banners for students to hold during the opening day game. One piece of cloth is 72 inches wide. The other is 90 inches wide. He wants to use all the cloth, and each square banner must be of equal width and as wide as possible.

   - ○ common multiples
   - ○ least common multiple
   - ○ common factors
   - ○ greatest common factor

   a. How wide should he cut the banners?

   b. How many banners can he cut?

5. **Dancers.** At Elena's dance recital her performance begins with a line of 48 dancers that perform in the dark with a black light that illuminates white clothing. All 48 dancers enter the stage in a straight line. Every 3rd dancer wears a white headband, every 5th dancer wears a white belt, and every 9th dancer wears a set of white gloves.

   - ○ common multiples
   - ○ least common multiple
   - ○ common factors
   - ○ greatest common factor

   a. If Elena is the 30th dancer, what accessories will she wear?

   b. Will any of the dancers wear all 3 accessories? If so, which one(s)?

   c. How many of each accessory will the dance teacher need to order?

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Lesson 18 Summary

If a problem requires dividing two whole numbers by the same whole number, solving it involves looking for a common factor. If it requires finding the *largest* number that can divide into the two whole numbers, we are looking for the *greatest common factor*.

Suppose we have 12 bagels and 18 muffins and want to make bags so each bag has the same combination of bagels and muffins. The common factors of 12 and 18 tell us possible number of bags that can be made.

The common factors of 18 are 1, 2, 3, and 6. For these numbers of bags, here are the number of bagels and muffins per bag.

- 1 bag: 12 bagels and 18 muffins
- 2 bags: 6 bagels and 9 muffins
- 3 bags: 4 bagels and 6 muffins
- 6 bags: 2 bagels and 3 muffins

We can see that the largest number of bags that can be made, 6, is the greatest common factor.

If a problem requires finding a number that is a multiple of two given numbers, solving it involves looking for a common multiple. If it requires finding the *first* instance the two numbers share a multiple, we are looking for the *least common multiple*.

Suppose forks are sold in boxes of 9 and spoons are sold in boxes of 15, and we want to buy an equal number of each. The multiples of 9 tell us how many forks we could buy, and the multiples of 15 tell us how many spoons we could buy, as shown here.

- Forks: 9, 18, 27, 36, 45, 54, 63, 72, 90. . .
- Spoons: 15, 30, 45, 60, 75, 90. . .

If we want as many forks as spoons, our options are 45, 90, 135, and so on, but the smallest number of utensils we could buy is 45, the least common multiple. This means buying 5 boxes of forks ($5 \cdot 9 = 45$) and 3 boxes of spoons ($3 \cdot 15 = 45$).